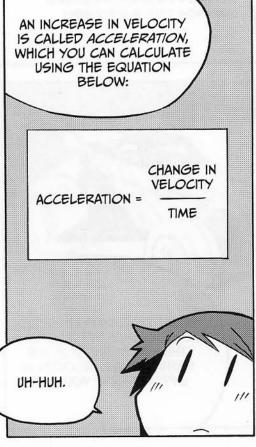




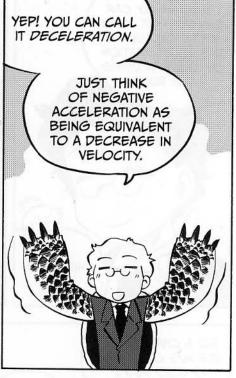
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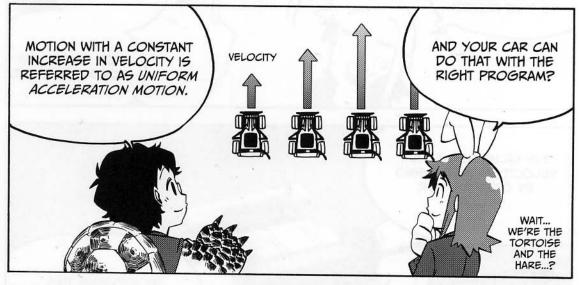


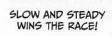


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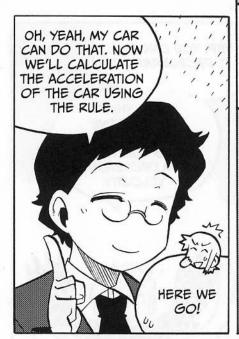


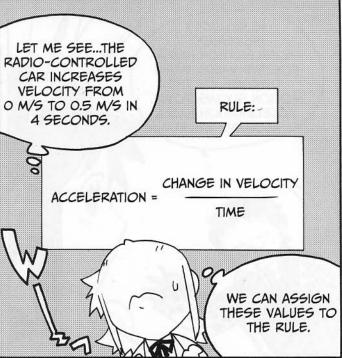




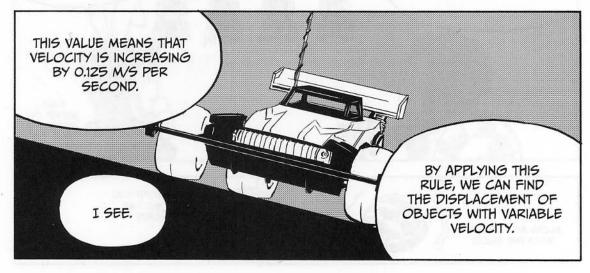


HEY! HOW'D YOU GET SO FAR AHEAD?









## LABORATORY

#### FINDING THE DISTANCE TRAVELED WHEN VELOCITY VARIES



Let's change the setting so as to steadily increase the velocity up to 0.5 m/s. Here's a quiz for you. Given that velocity has attained 0.5 m/s in four seconds, how far has the radio-controlled car moved?



Hmm . . . starting at 0 m/s, the peak velocity is 0.5 m/s. So let me calculate, assuming the average speed, 0.25 m/s, for the velocity. 0.25 m/s × 4 s  $= 1 \, \text{m}!$ 



That's right! You are so sharp. But can you explain why you can get the right answer with that calculation?



Uhm . . . remember, teaching me is your job, Nonomura-kun!



Ha ha, true enough. Before giving you a direct answer, let me explain how we can find the distance traveled when the velocity varies. When velocity is constant, we've learned that the distance traveled can be found by calculating the expression (speed  $\times$  time). Now, given that d m (meters) represents the distance traveled in t s (seconds) and the constant velocity is v m/s, then distance = speed × time can be expressed in the following equation:

d = vt

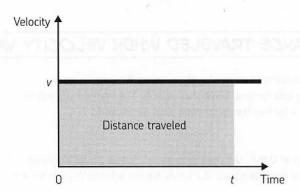


Well, duh!





If you plot that relationship with velocity on the vertical axis and time on the horizontal axis, you get the following graph.



The shaded area represents the distance traveled. This chart is commonly referred to as a *v-t graph*, as it graphs velocity and time. That's the area of a rectangle having a horizontal length of *t* and a vertical length of *v*.



I see. It seems a little strange that an area represents a distance.



The area here is not a typical geometric area—this is a graph, like the ones you've seen in math class. The area of a geometric rectangle might be measured in square meters ( $m^2$ ). But in our example, the units are time (seconds) for the horizontal axis and velocity (m/s) for the vertical axis. So the product of these two is equal to  $s \times m/s = m$ . That's our unit for distance.



It's easy to find a distance when an object goes at a constant speed. But what about finding the distance when the speed is variable?



The only tool available to us is this equation:

distance = speed × time



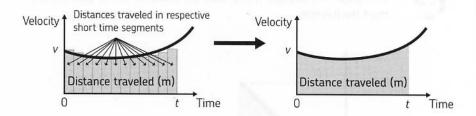
So we can divide the time into segments to create a lot of "small rectangles" and then calculate distances respectively, assuming a constant velocity for each time segment.



What do you mean?



Look at the chart on the left below.



So we can find the area of each slender rectangle created by dividing time into short segments, and then adding up the areas to find the distance traveled.



It bothers me that those little rectangles won't exactly fit the graph. Wouldn't they bring about errors?



I see your concern. Then we can sub-divide the rectangles into smaller segments. By repeating division into even smaller segments until everything fits as shown in the chart on the right above, the distance we get becomes more and more precise.



Well, I guess so . . . if you could do that . . .



If we divided them into infinitely slender rectangles, we'd find exactly how far the object has moved. After all, the ultimate answer we get by dividing distance = speed × time into short time segments is the area created under a v-t graph. That's how we can find the distance traveled by finding the corresponding area. In summary,

distance traveled = area under a v-t graph

Just like that.\*

<sup>\*</sup> Students of calculus may notice that this process of finding an area under a graph is identical to integration.



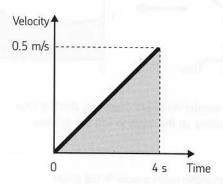
Now, keeping in mind what we've learned so far, let's examine the reason why the distance you got intuitively is the right answer.



All right!



Your original calculation is the same as calculating an area on a velocitytime graph. The example with a radio-controlled car can be plotted into a chart like this one.



The area under the graph, as obtained from the rule for the area of a triangle, is as follows:

 $\frac{1}{2}$  × base (time) × height (max velocity) =  $\frac{1}{2}$  × 4 s × 0.5 m/s = 1 m

This represents the distance traveled.



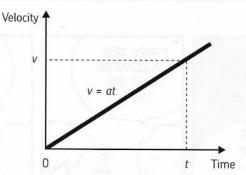
We got 1 meter for the answer, just as we should.



Let's find a general expression for the distance traveled, rather than using specific numeric values. Assuming velocity to be v and acceleration to be a, the relationship between the velocity and time for uniform accelerated motion is v = at.

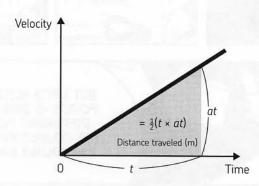


That can be plotted into a v-t graph, as shown below.



Let's assume *d* is the distance traveled in time *t*; its value should be equivalent to the area of a triangle with a base of t and height of at (which equals the final velocity of the object).

$$d = \frac{1}{2}at^2$$



You see?

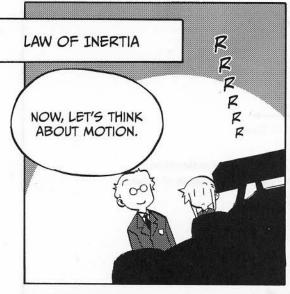


Ummmm . . . oh, I see how that works! The value we get by calculating  $\frac{1}{2}$  ×  $0.125 \text{ m/s}^2 \times (4 \text{ s})^2 = 1 \text{ m. As it should be!}$ 



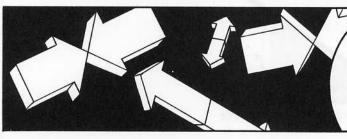
Now, Ninomiya-san, you can also calculate a distance traveled in uniform accelerated motion not by intuition but by the proper method.

#### NEWTON'S FIRST AND SECOND LAWS

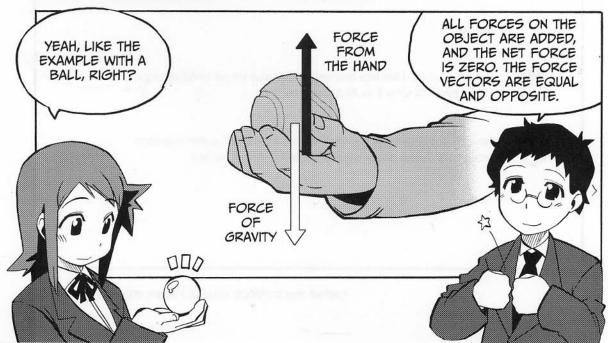








BUT LET'S NOTE THAT THE FORCE IS ZERO BECAUSE DIFFERENT FORCES ON THE OBJECT ARE ACTUALLY CANCELING EACH OTHER.



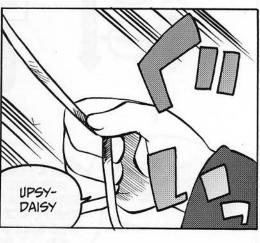




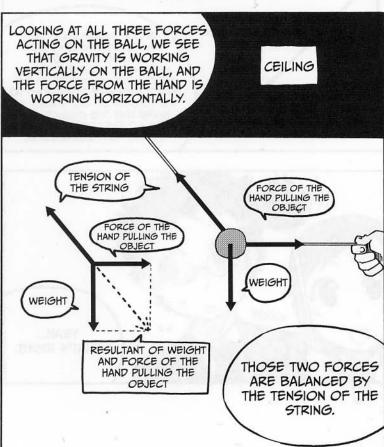








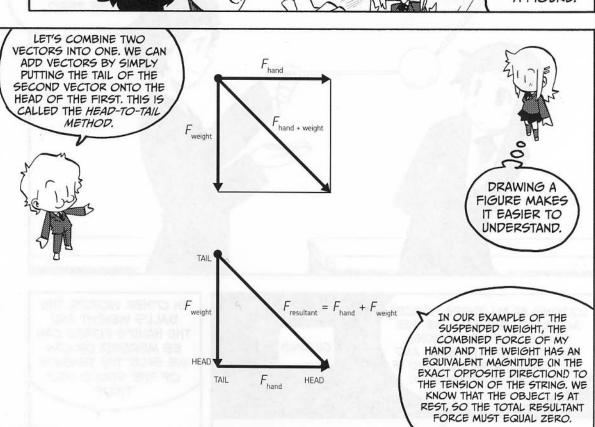




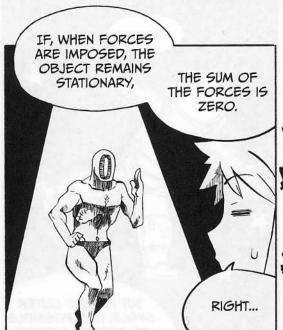


LAW OF INERTIA 61



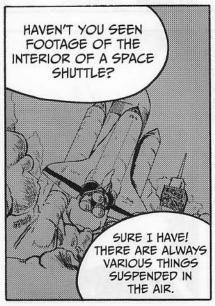




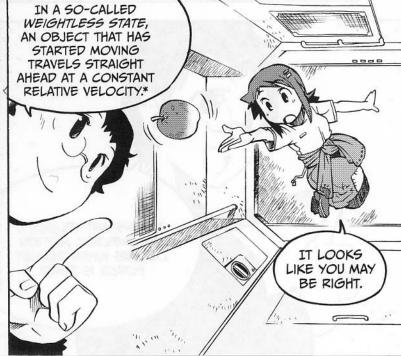




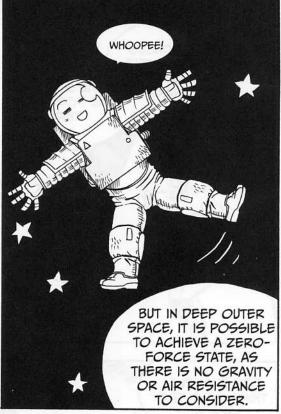












YES, INDEED! IN THAT CASE, YOU MEAN, WE COULD KEEP MOVING FOREVER, EVEN WITH NO FORCE IMPOSED?

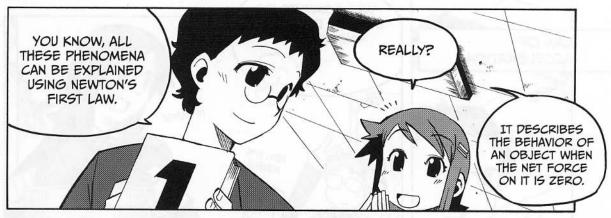


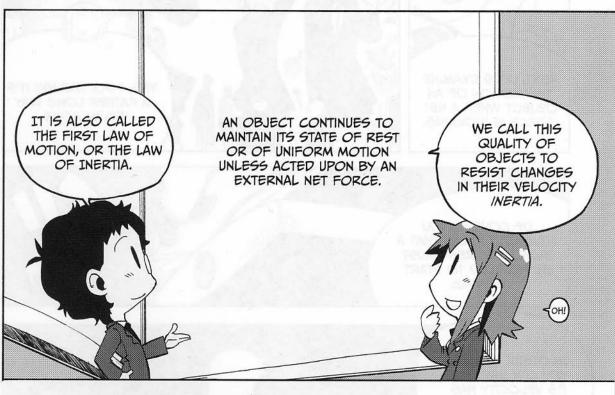




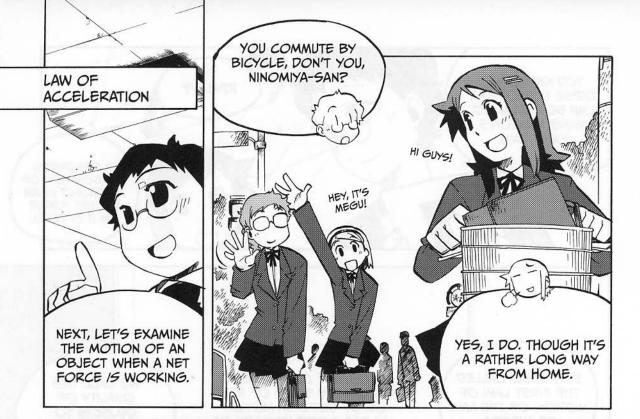


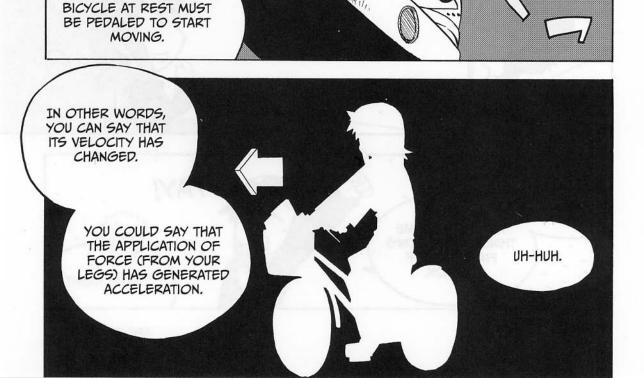






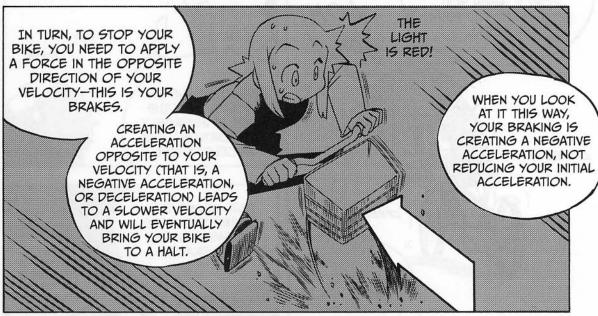






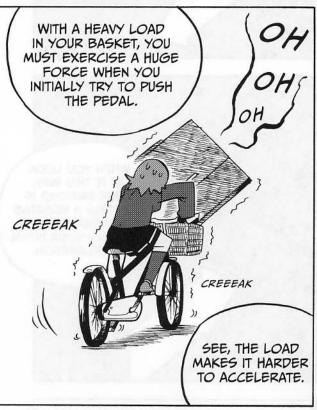
OF COURSE, YOU INTUITIVELY KNOW THAT A

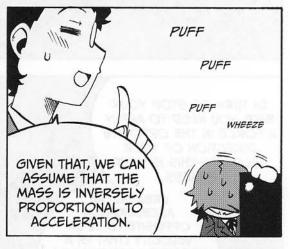




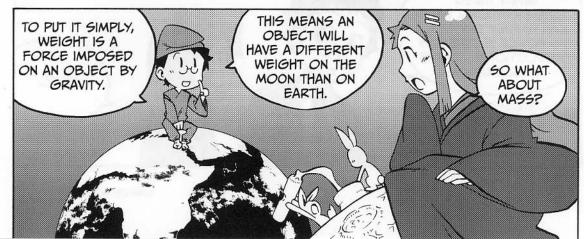


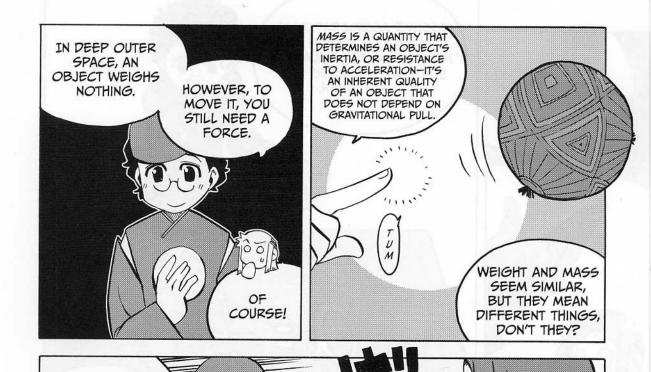












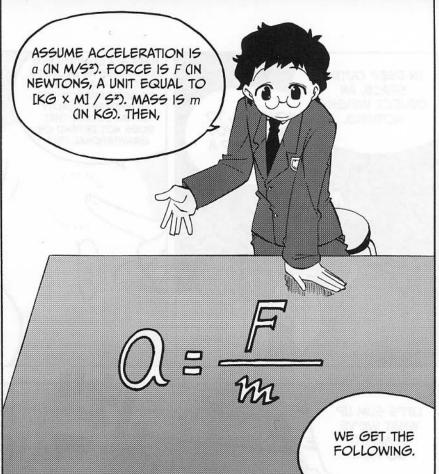
UH, OKAY.

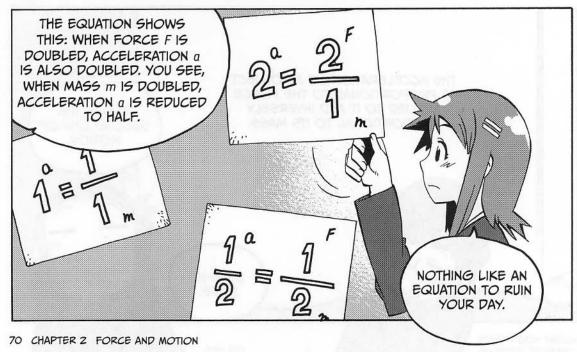
LET'S SUM UP WHAT WE'VE

LEARNED

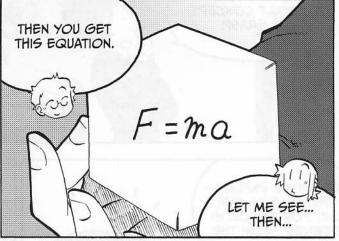




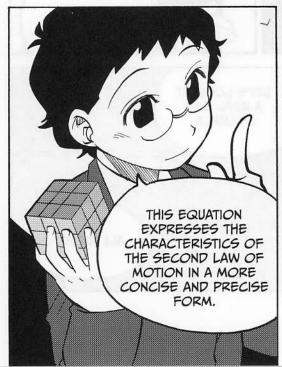












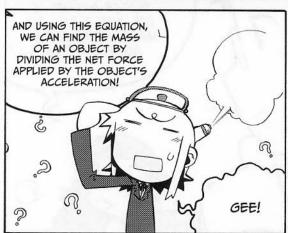


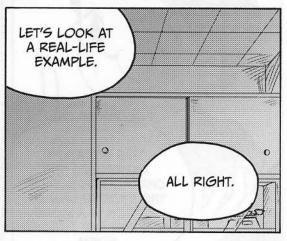
LAW OF ACCELERATION 71











# LABORATORY

### FINDING THE PRECISE VALUE OF A FORCE



Earlier, we pushed each other while we were on roller blades. Let's say that I captured our motion on video.



I didn't realize you were taping us!



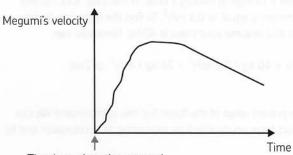
Oh, that's just the scenario I'm setting up.



Jeez, don't scare me. How does that relate to the second law of motion?



Suppose I have analyzed the video, and I've created a v-t graph of your motion.



The time when they started pushing each other

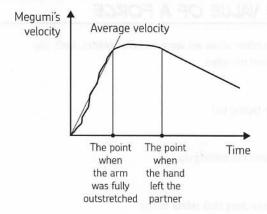


We can see that velocity increases sharply from zero, which must be when I'm at rest, and then drops gradually after that. But the initial increase in velocity is wobbly.





In a case like this, it may be a good idea to draw a line segment that represents the average increase in velocity. In other words, we'll simplify the scenario to assume this is a case of uniform acceleration.





I see.



You can find acceleration by calculating the change in acceleration over time—acceleration = change in velocity / time. In this case, let's assume that your acceleration is equal to 0.6 m/s<sup>2</sup>. To find the force I applied to your hands, let's also assume your mass is 40 kg, Ninomiya-san.

$$F = ma = 40 \text{ kg} \times 0.6 \text{ m/s}^2 = 24 \text{ kg} \times \text{m/s}^2$$
, or 24N



We've found the precise value of the force! So, this is important! We can measure the exact force on an object by measuring its acceleration and its mass.



Now, if you know that I weigh 60 kg, can you predict my acceleration, due to the application of an equal and opposite 24N of force?



Oh, I see. We're combining the second and third laws of motion.  $F_{\text{Megumi}}$  must equal  $F_{\text{Ryota}}$ . Since F = ma, we know that F / m = a. In your case, that's 24N / 60 kg, or 0.4 m/s². So we can use these laws to predict the movement of objects. Neat!





