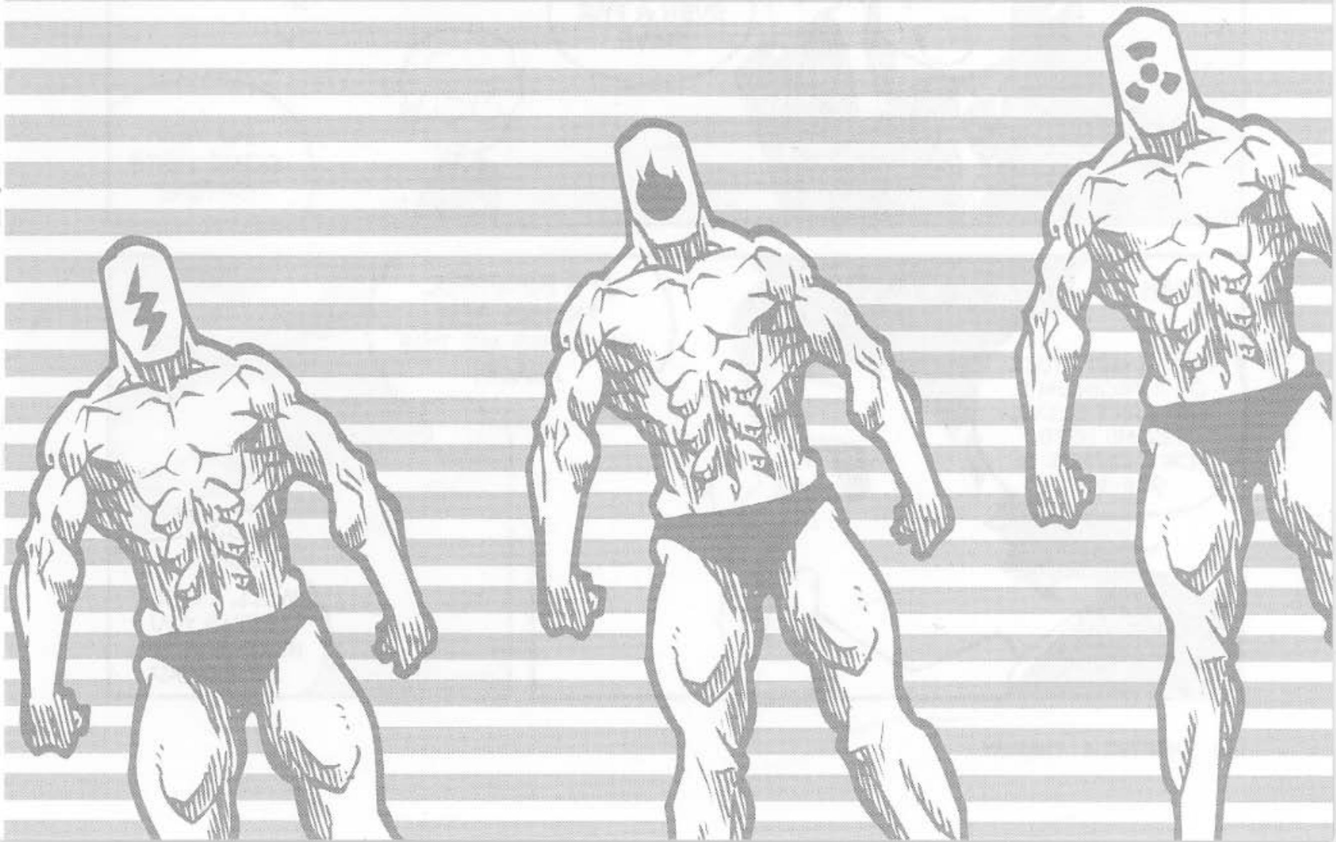
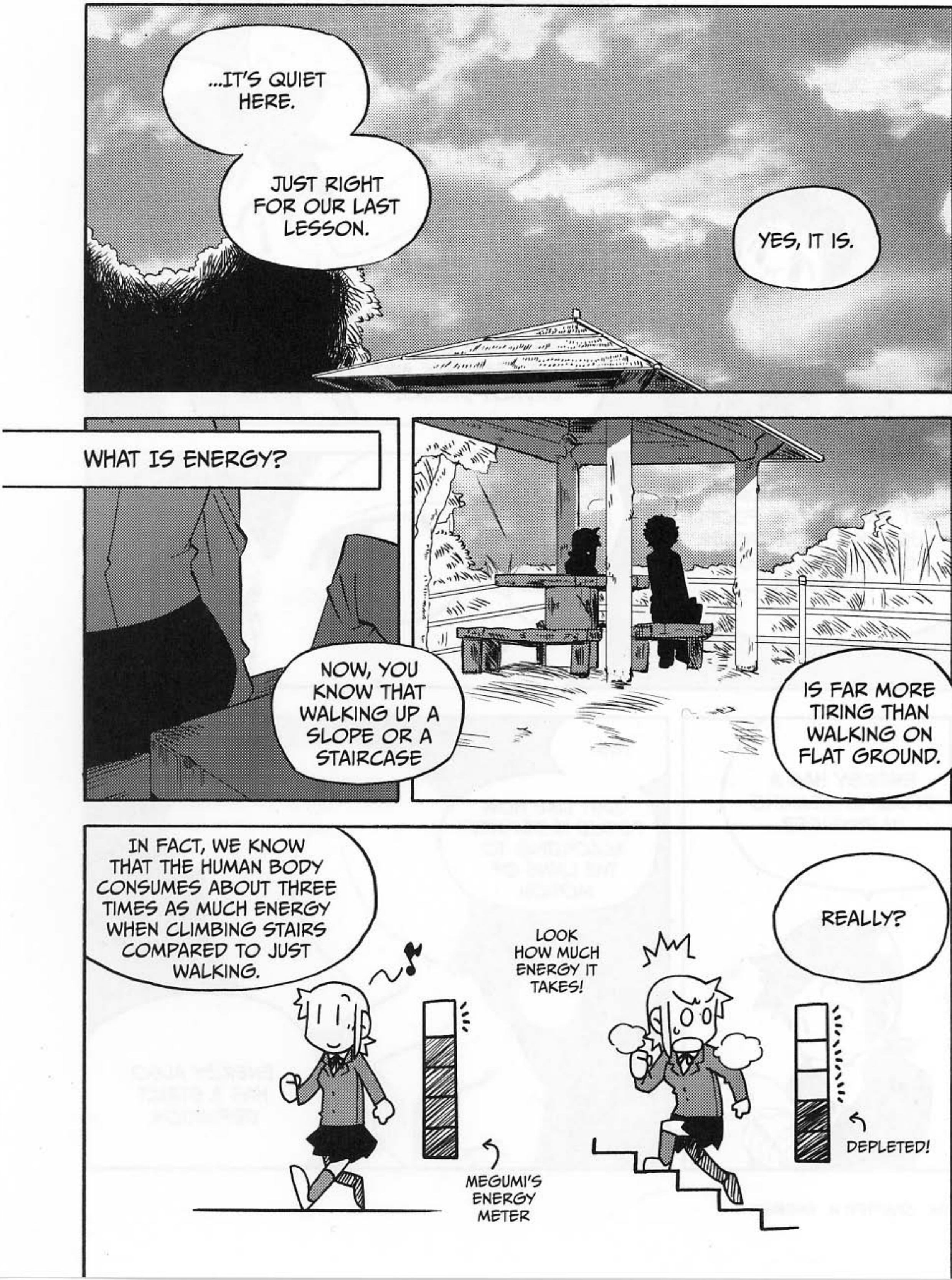
THE MANGA GUIDE™ TO PHYSICS

4

ENERGY

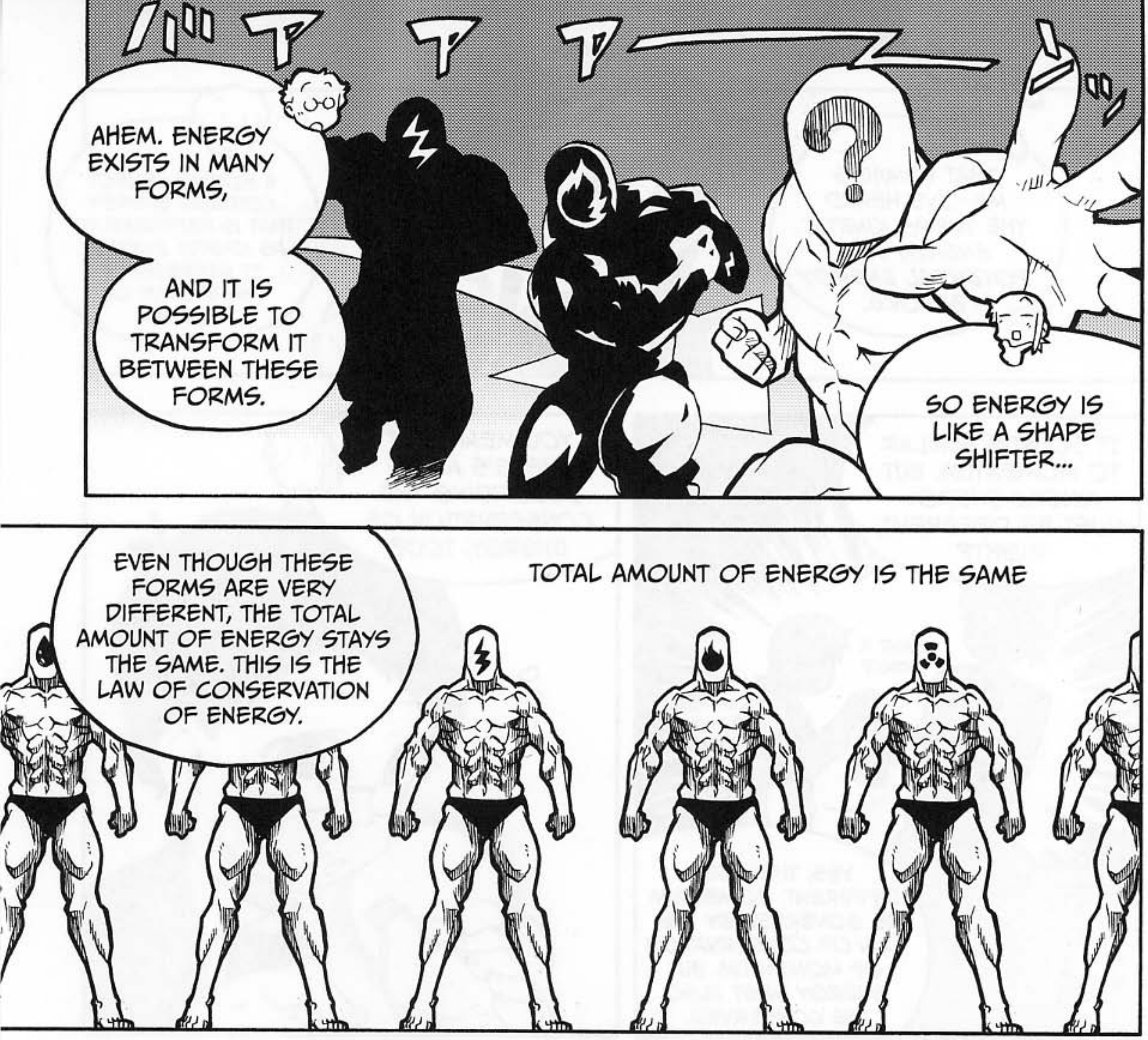


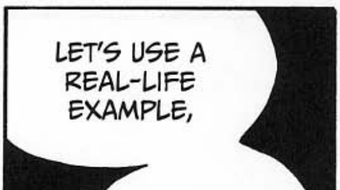
WORK AND ENERGY

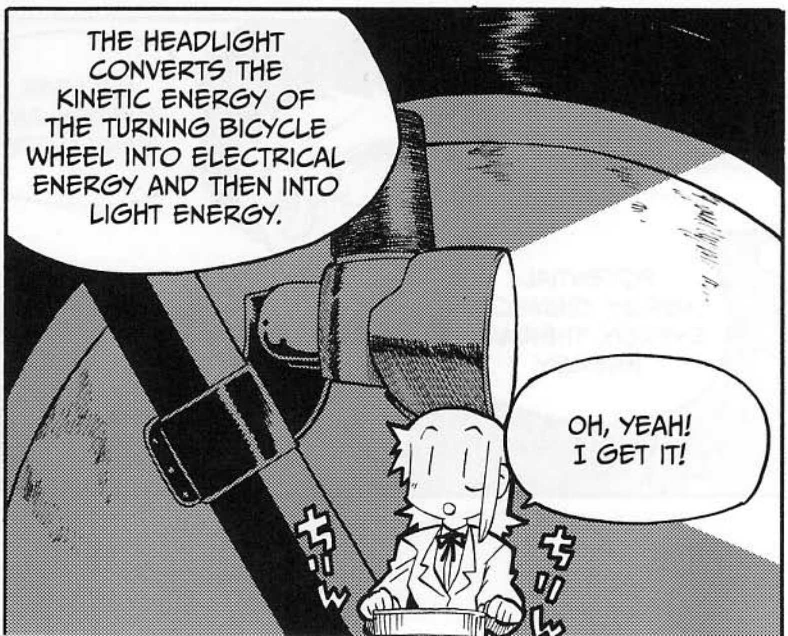

...IT'S QUIET HERE.
JUST RIGHT FOR OUR LAST LESSON.
YES, IT IS.
WHAT IS ENERGY?
NOW, YOU KNOW THAT WALKING UP A SLOPE OR A STAIRCASE
IS FAR MORE TIRING THAN WALKING ON FLAT GROUND.
IN FACT, WE KNOW THAT THE HUMAN BODY CONSUMES ABOUT THREE TIMES AS MUCH ENERGY WHEN CLIMBING STAIRS COMPARED TO JUST WALKING.
LOOK HOW MUCH ENERGY IT TAKES!
REALLY?
MEGUMI'S ENERGY METER
DEPLETED!

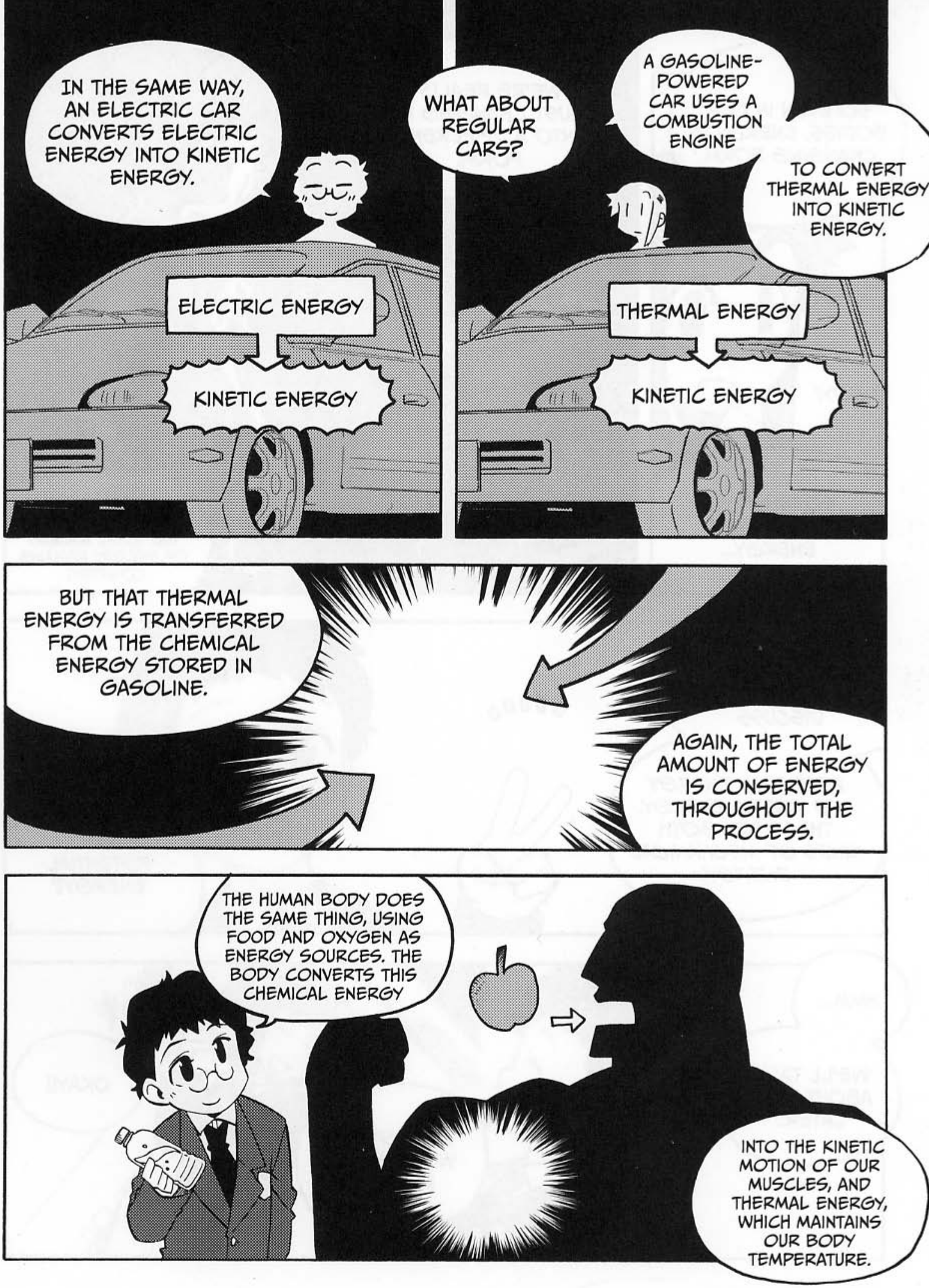

BUT WE SEE THE TERM *ENERGY* ALL OVER THE PLACE, DON'T WE?
YEAH! LIKE ENERGY-EFFICIENT CARS, GREEN ENERGY, AND ENERGY DRINKS!
ENERGY IS A WORD A LOT LIKE *FORCE*. PEOPLE USE THE TERM RATHER LOOSELY TO DESCRIBE THINGS, BUT...
WAIT! YOU MEAN...
ENERGY HAS A SPECIFIC MEANING IN PHYSICS?
YES.
JUST LIKE HOW FORCE IS DEFINED ACCORDING TO THE LAWS OF MOTION,
ENERGY ALSO HAS A STRICT DEFINITION.


GLUG GULP
THAT REMINDS ME—I'VE HEARD THE TERMS *KINETIC ENERGY* AND *POTENTIAL ENERGY* BEFORE.
AHHH!
A MOVING OBJECT CONTAINS ENERGY THAT IS REFERRED TO AS *KINETIC ENERGY*. IT REPRESENTS THE ENERGY OF MOTION.
IT SOUNDS SIMILAR TO MOMENTUM. BUT KINETIC ENERGY MUST BE DIFFERENT, RIGHT?
WANT A DRINK?
ど、どうも…
YES, THEY ARE DIFFERENT. MOMENTUM IS GOVERNED BY THE LAW OF CONSERVATION OF MOMENTUM. BUT ENERGY MUST ALSO BE CONSERVED.
YOU MEAN THAT THERE'S A LAW DESCRIBING THE CONSERVATION OF ENERGY, TOO?
YES. ENERGY CAN TAKE MANY FORMS, THOUGH. THERE'S KINETIC ENERGY,
POTENTIAL ENERGY, CHEMICAL ENERGY, THERMAL ENERGY,
NUCLEAR ENERGY, AND MANY MORE.
YUCK!
DON'T YOU LIKE IT?
UH, ARE YOU OKAY?


AHEM. ENERGY EXISTS IN MANY FORMS,
AND IT IS POSSIBLE TO TRANSFORM IT BETWEEN THESE FORMS.
SO ENERGY IS LIKE A SHAPE SHIFTER...
EVEN THOUGH THESE FORMS ARE VERY DIFFERENT, THE TOTAL AMOUNT OF ENERGY STAYS THE SAME. THIS IS THE LAW OF CONSERVATION OF ENERGY.
TOTAL AMOUNT OF ENERGY IS THE SAME

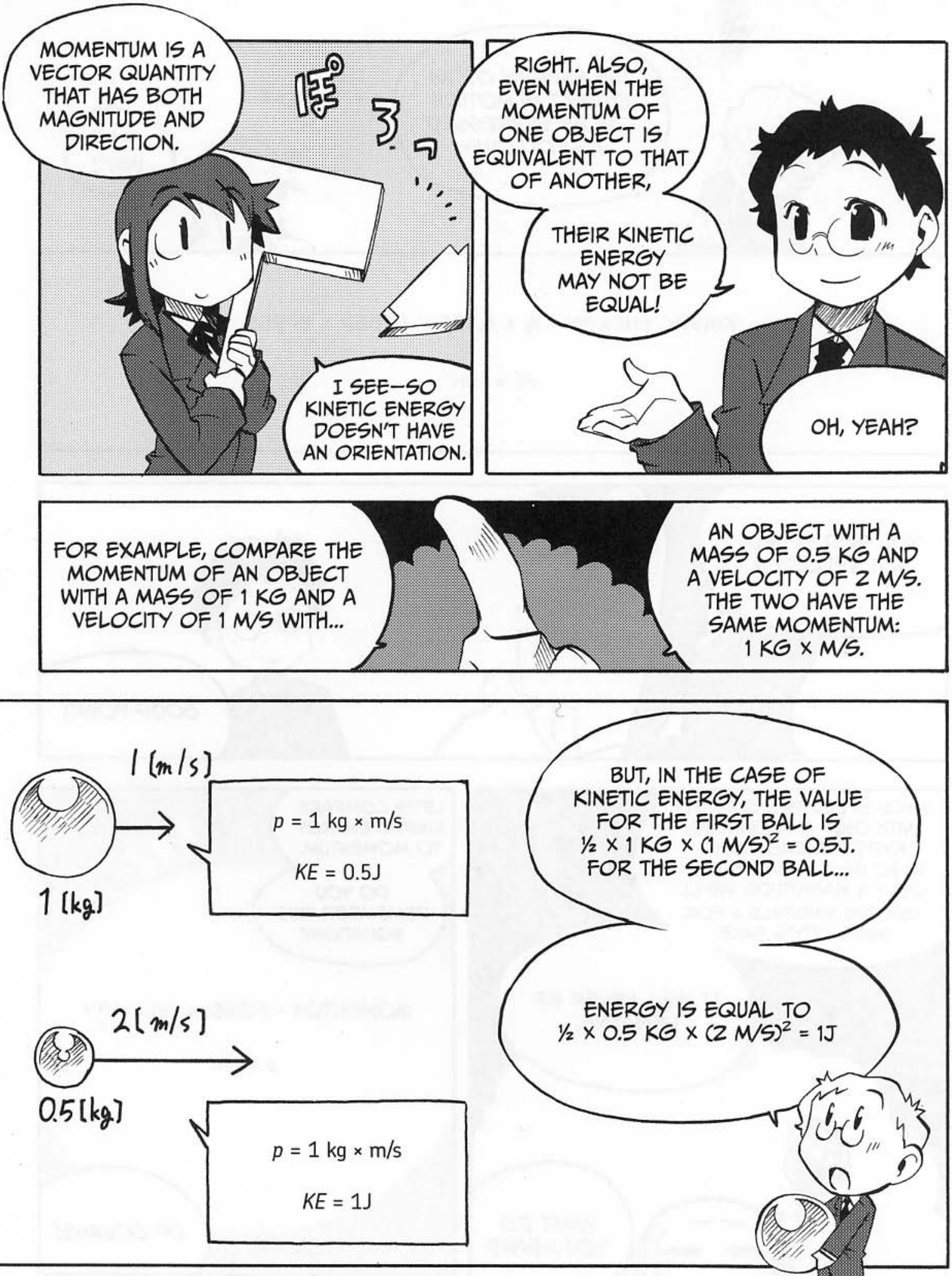

LET'S USE A REAL-LIFE EXAMPLE,
LIKE A HEADLIGHT ON A BICYCLE.

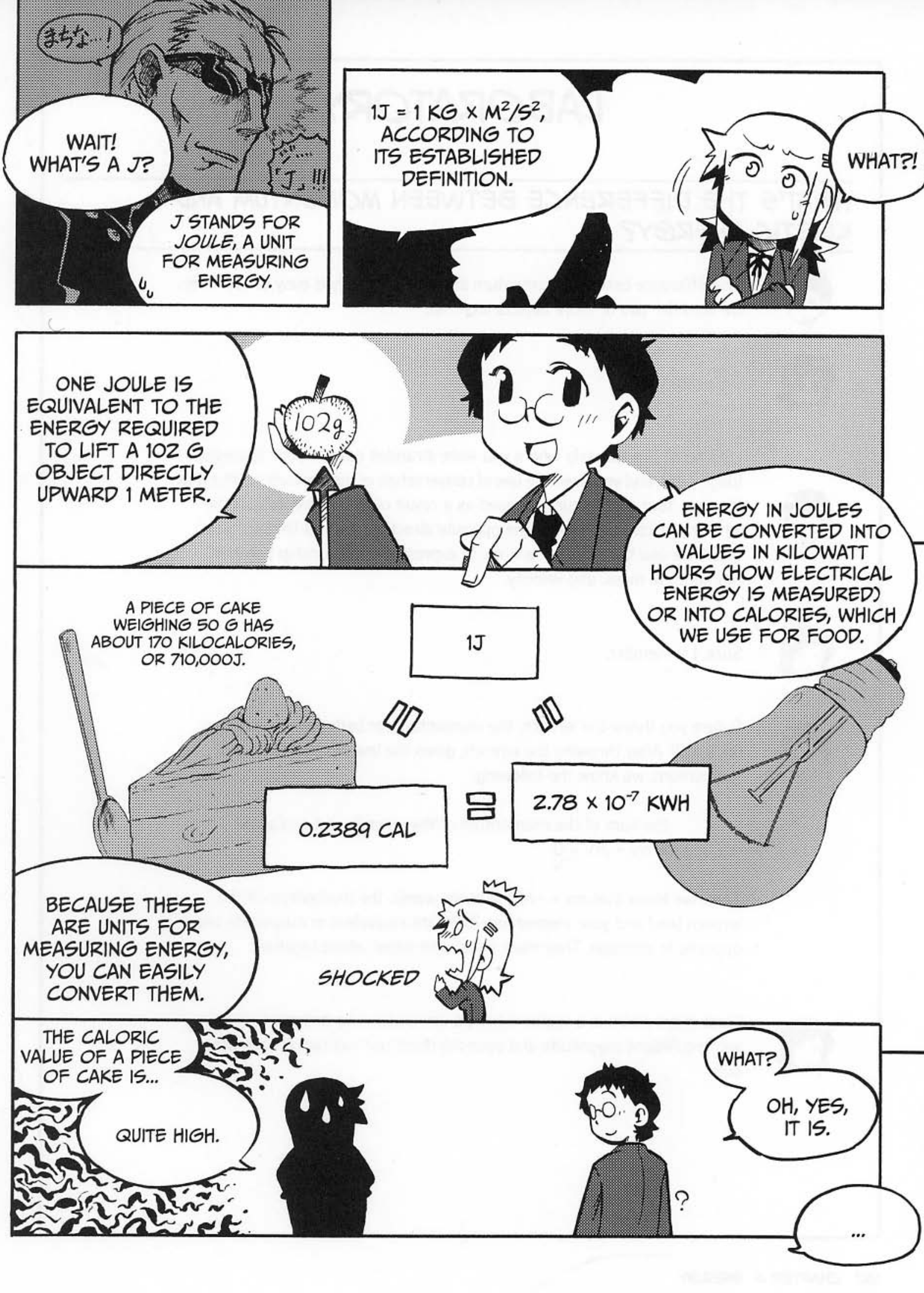

THE HEADLIGHT CONVERTS THE KINETIC ENERGY OF THE TURNING BICYCLE WHEEL INTO ELECTRICAL ENERGY AND THEN INTO LIGHT ENERGY.
OH, YEAH! I GET IT!


IN THE SAME WAY, AN ELECTRIC CAR CONVERTS ELECTRIC ENERGY INTO KINETIC ENERGY.
ELECTRIC ENERGY
KINETIC ENERGY
WHAT ABOUT REGULAR CARS?
A GASOLINE-POWERED CAR USES A COMBUSTION ENGINE
TO CONVERT THERMAL ENERGY INTO KINETIC ENERGY.
THERMAL ENERGY
KINETIC ENERGY
BUT THAT THERMAL ENERGY IS TRANSFERRED FROM THE CHEMICAL ENERGY STORED IN GASOLINE.
AGAIN, THE TOTAL AMOUNT OF ENERGY IS CONSERVED, THROUGHOUT THE PROCESS.
THE HUMAN BODY DOES THE SAME THING, USING FOOD AND OXYGEN AS ENERGY SOURCES. THE BODY CONVERTS THIS CHEMICAL ENERGY
INTO THE KINETIC MOTION OF OUR MUSCLES, AND THERMAL ENERGY, WHICH MAINTAINS OUR BODY TEMPERATURE.

SO EVEN IN OUR BODIES, ENERGY IS CHANGING FORM.
BOY!
SO WHEN WE "CONSUME" ENERGY...
WE'RE REALLY JUST CHANGING IT INTO A DIFFERENT FORM.
ENERGY IS ALWAYS CIRCULATING, BUT THE TOTAL AMOUNT OF ENERGY REMAINS CONSTANT.
BUT LET'S GET A LITTLE LESS ABSTRACT AND DISCUSS
POTENTIAL ENERGY AND KINETIC ENERGY. THESE ARE BOTH KINDS OF *MECHANICAL ENERGY.*
POTENTIAL ENERGY?
HMM...
WE'LL TALK ABOUT THAT LATER.
LET'S START WITH KINETIC ENERGY.
OKAY!!


THE ENERGY OF AN OBJECT IN MOTION CAN BE EXPRESSED AS FOLLOWS:
BUT WAIT!
KINETIC ENERGY = ½ × MASS × SPEED × SPEED
$KE = \frac{1}{2}mv^2$
YOU SAID SPEED, NOT VELOCITY!
GOOD POINT!
SINCE SPEED IS A QUANTITY WITH ONLY A MAGNITUDE, KINETIC ENERGY MUST ALSO BE A QUANTITY WITH ONLY A MAGNITUDE. WE'LL USE THE VARIABLE v FOR SIMPLICITY'S SAKE.
IT WILL NEVER BE NEGATIVE.
WHAT DO YOU MEAN?
LET'S COMPARE KINETIC ENERGY TO MOMENTUM.
DO YOU REMEMBER THIS EQUATION?
MOMENTUM = MASS × VELOCITY
$p = mv$
OF COURSE!


MOMENTUM IS A VECTOR QUANTITY THAT HAS BOTH MAGNITUDE AND DIRECTION.
RIGHT. ALSO, EVEN WHEN THE MOMENTUM OF ONE OBJECT IS EQUIVALENT TO THAT OF ANOTHER,
THEIR KINETIC ENERGY MAY NOT BE EQUAL!
I SEE—SO KINETIC ENERGY DOESN'T HAVE AN ORIENTATION.
OH, YEAH?
FOR EXAMPLE, COMPARE THE MOMENTUM OF AN OBJECT WITH A MASS OF 1 KG AND A VELOCITY OF 1 M/S WITH...
AN OBJECT WITH A MASS OF 0.5 KG AND A VELOCITY OF 2 M/S. THE TWO HAVE THE SAME MOMENTUM: 1 KG × M/S.
1 [m/s]
1 [kg]
$p = 1$ kg × m/s
$KE = 0.5$J
2 [m/s]
0.5 [kg]
$p = 1$ kg × m/s
$KE = 1$J
BUT, IN THE CASE OF KINETIC ENERGY, THE VALUE FOR THE FIRST BALL IS ½ × 1 KG × (1 M/S)^2 = 0.5J. FOR THE SECOND BALL...
ENERGY IS EQUAL TO ½ × 0.5 KG × (2 M/S)^2 = 1J


まちな…!
WAIT! WHAT'S A J?
「J」!!!
J STANDS FOR JOULE, A UNIT FOR MEASURING ENERGY.
1J = 1 KG × M²/S², ACCORDING TO ITS ESTABLISHED DEFINITION.
WHAT?!
ONE JOULE IS EQUIVALENT TO THE ENERGY REQUIRED TO LIFT A 102 G OBJECT DIRECTLY UPWARD 1 METER.
102g
ENERGY IN JOULES CAN BE CONVERTED INTO VALUES IN KILOWATT HOURS (HOW ELECTRICAL ENERGY IS MEASURED) OR INTO CALORIES, WHICH WE USE FOR FOOD.
A PIECE OF CAKE WEIGHING 50 G HAS ABOUT 170 KILOCALORIES, OR 710,000J.
1J
0.2389 CAL
=
2.78 × 10⁻⁷ KWH
BECAUSE THESE ARE UNITS FOR MEASURING ENERGY, YOU CAN EASILY CONVERT THEM.
SHOCKED
THE CALORIC VALUE OF A PIECE OF CAKE IS...
QUITE HIGH.
WHAT?
OH, YES, IT IS.
?
...

LABORATORY

WHAT'S THE DIFFERENCE BETWEEN MOMENTUM AND KINETIC ENERGY?

The difference between momentum and kinetic energy is easy to see when we consider two or more objects together.

Oh, yeah?

Let's recall the scenario where you were stranded outside your spaceship (page 126), and you used the law of conservation of momentum to return to the ship. Your momentum changed as a result of the momentum of the wrench, which you threw in the opposite direction. And, as I'm sure you recall, we use the equation $p = mv$ to express the relationship between momentum, mass, and velocity.

Sure, I remember.

Before you threw the wrench, the momentum for both objects was zero (as $v = 0$). After throwing the wrench, given the law of conservation of momentum, we know the following:

the sum of the momentum of the wrench and astronaut
$= mv + MV = 0$

Thus, we know that $mv = -MV$. In other words, the momentum of the wrench (mv) and your momentum (MV) are equivalent in magnitude and opposite in direction. They must equal zero when added together.

Since momentum is a vector, it has an orientation! So two momentums with equivalent magnitude and opposite directions will cancel each other out.

Now, let's think about the kinetic energy of the wrench and that of the astronaut. Before throwing the wrench, both are stationary, and the momentum is zero for both objects. After throwing the wrench, the sum of the energy of the two objects in motion is *not* zero:

$$KE_{\text{wrench}} + KE_{\text{astronaut}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}MV^2 > 0$$

But you said energy is always conserved!

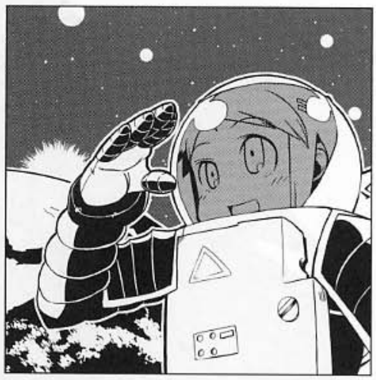
This kinetic energy was generated when you threw the tool. Consider the law of conservation of energy—the amount of energy lost in your body should be the same as the amount of kinetic energy gained in these two objects.

Well, okay.

While it's difficult to accurately measure the energy expended by the human body, we can say that it's possible to determine a decrease of energy in the body by finding the energy transferred by that body.

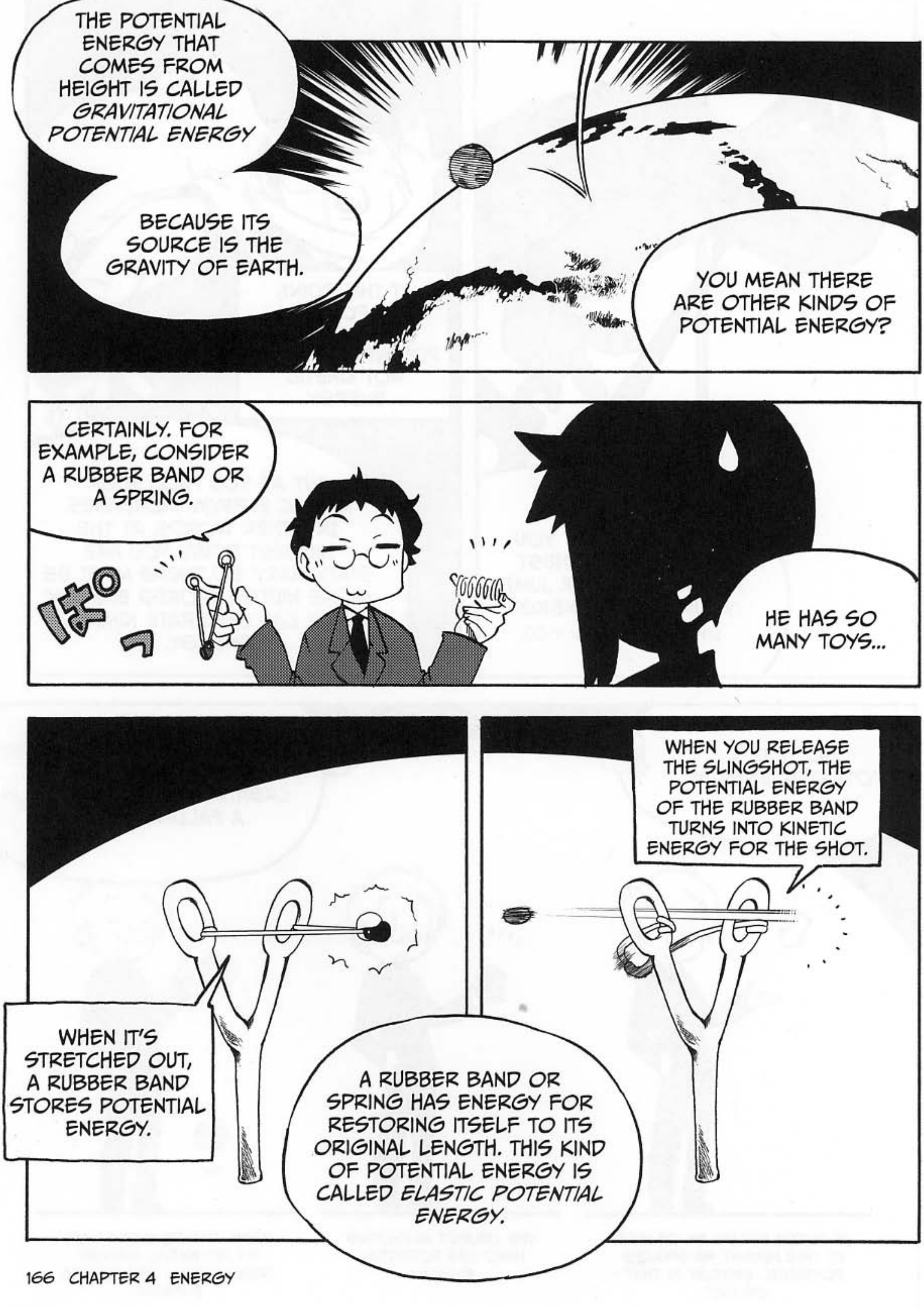
In other words, I know that my body has lost at least as much energy as I have gained in the objects I've thrown, right?

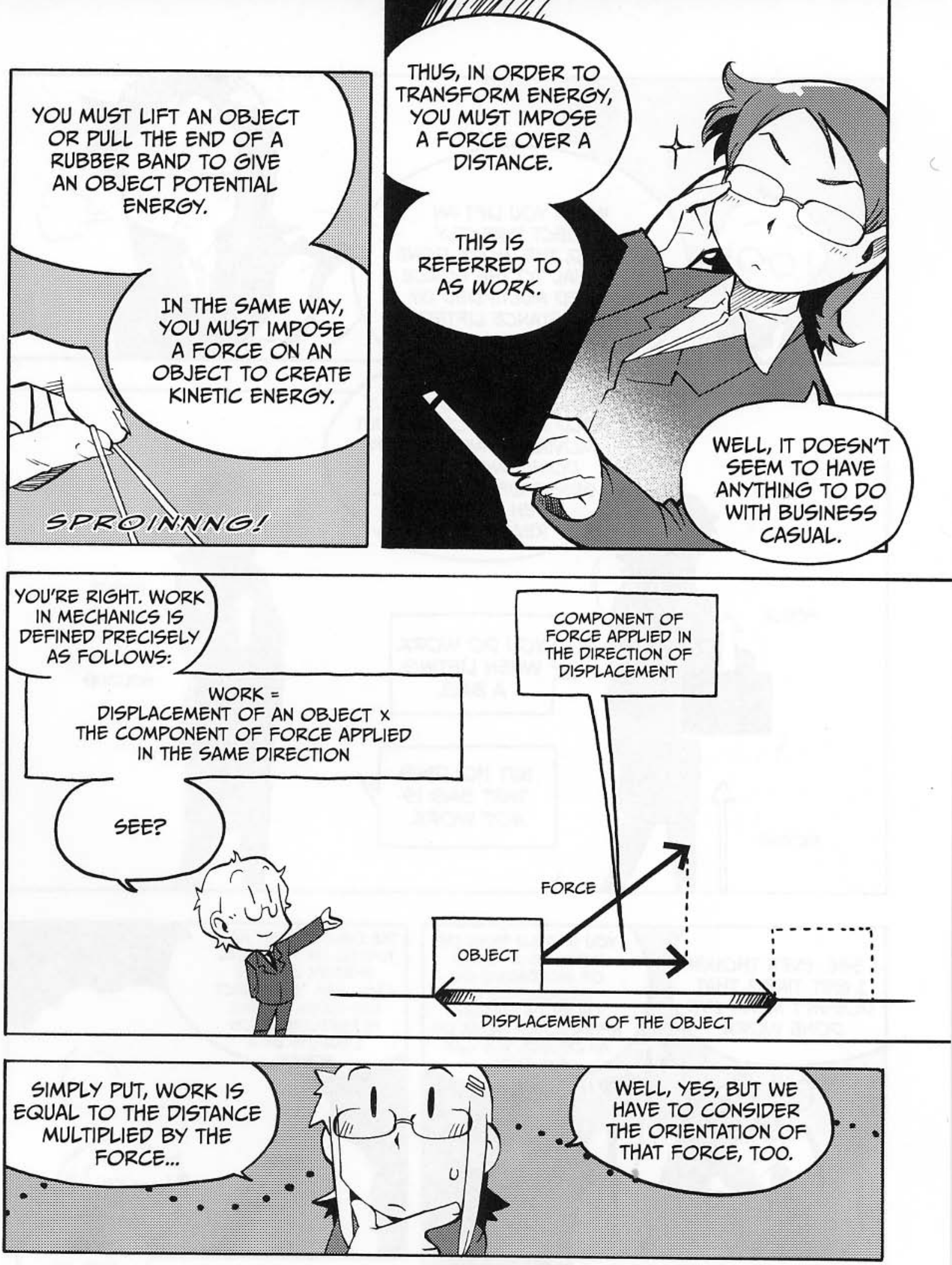
Yes, that's it. Now you need to remember, we must keep in mind the differences between energy and momentum.

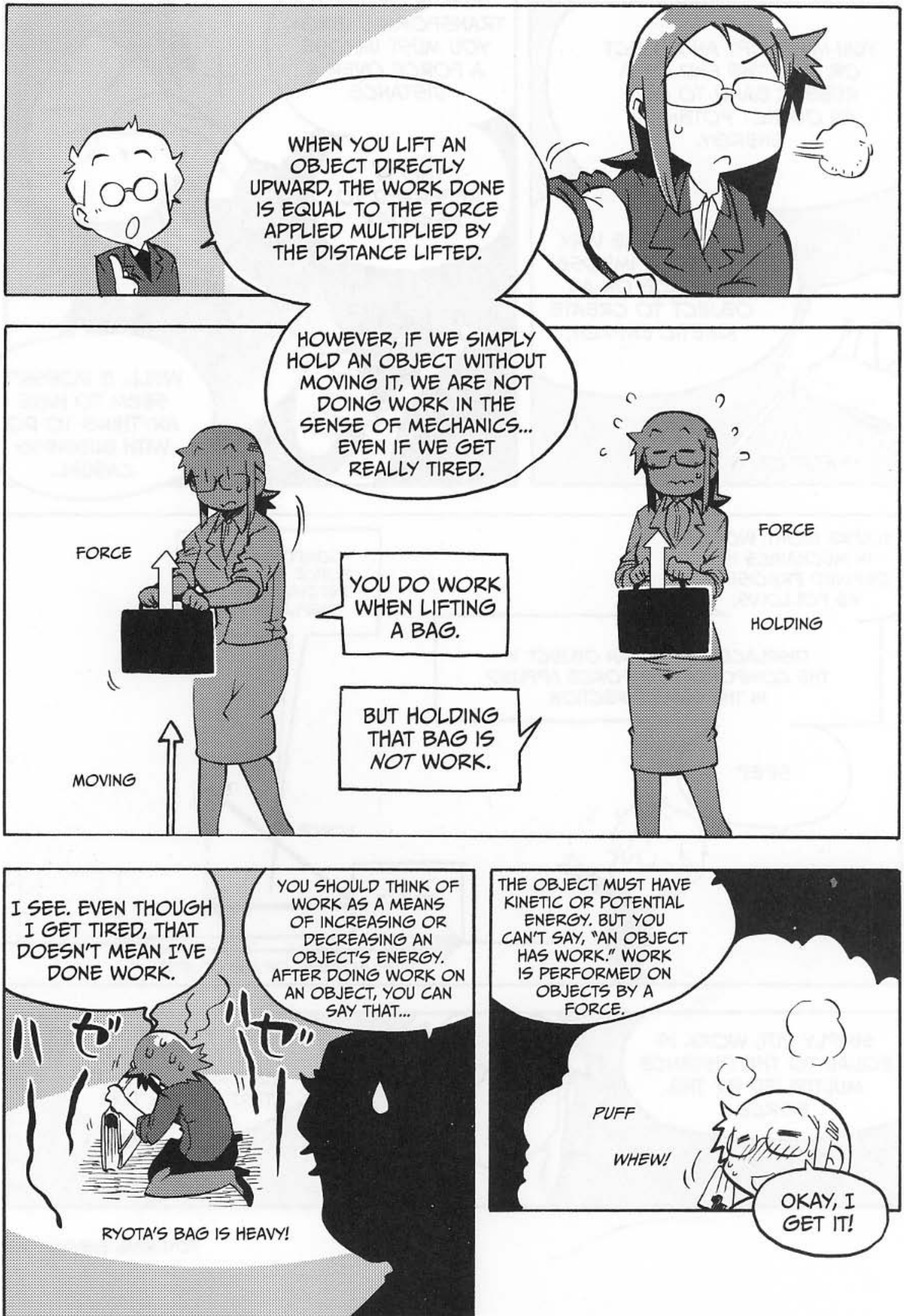



POTENTIAL ENERGY
EARLIER, I MENTIONED THAT MECHANICAL ENERGY INCLUDES KINETIC ENERGY AND POTENTIAL ENERGY.
YOU CAN THINK OF *POTENTIAL ENERGY* AS THE ENERGY OF POSITION.
WHAT DOES THAT MEAN?
WELL,
POTENTIAL REFERS TO THE STORED ABILITY TO DO WORK.
Potential
SO DOES POTENTIAL ENERGY MEAN STORED ENERGY?
LET'S USE YOUR HIGH JUMP AS AN EXAMPLE.


AT THE MOMENT YOU REACH THE HIGHEST POSITION IN YOUR JUMP, YOUR KINETIC ENERGY DISAPPEARS (v = 0).
AT THIS POINT, YOU POSSESS GRAVITATIONAL POTENTIAL ENERGY, NOT KINETIC ENERGY.
BUT AS YOU FALL, YOUR KINETIC ENERGY INCREASES. IN OTHER WORDS, AT THE HIGHEST POINT, YOU ARE STATIONARY. SO THERE MUST BE SOME HIDDEN STORED ENERGY THAT CAN GENERATE KINETIC ENERGY.
SO THAT'S POTENTIAL ENERGY.
YES, THE POTENTIAL ENERGY OF A PARTICULAR HEIGHT CREATES KINETIC ENERGY IN A FALLING OBJECT.
IF RYOTA HOLDS AN OBJECT AT THIS HEIGHT, HE STORES POTENTIAL ENERGY IN THAT OBJECT.
THE OBJECT IN RYOTA'S HAND HAS POTENTIAL ENERGY.
WHEN THE OBJECT FALLS, ITS POTENTIAL ENERGY TRANSFORMS INTO KINETIC ENERGY.

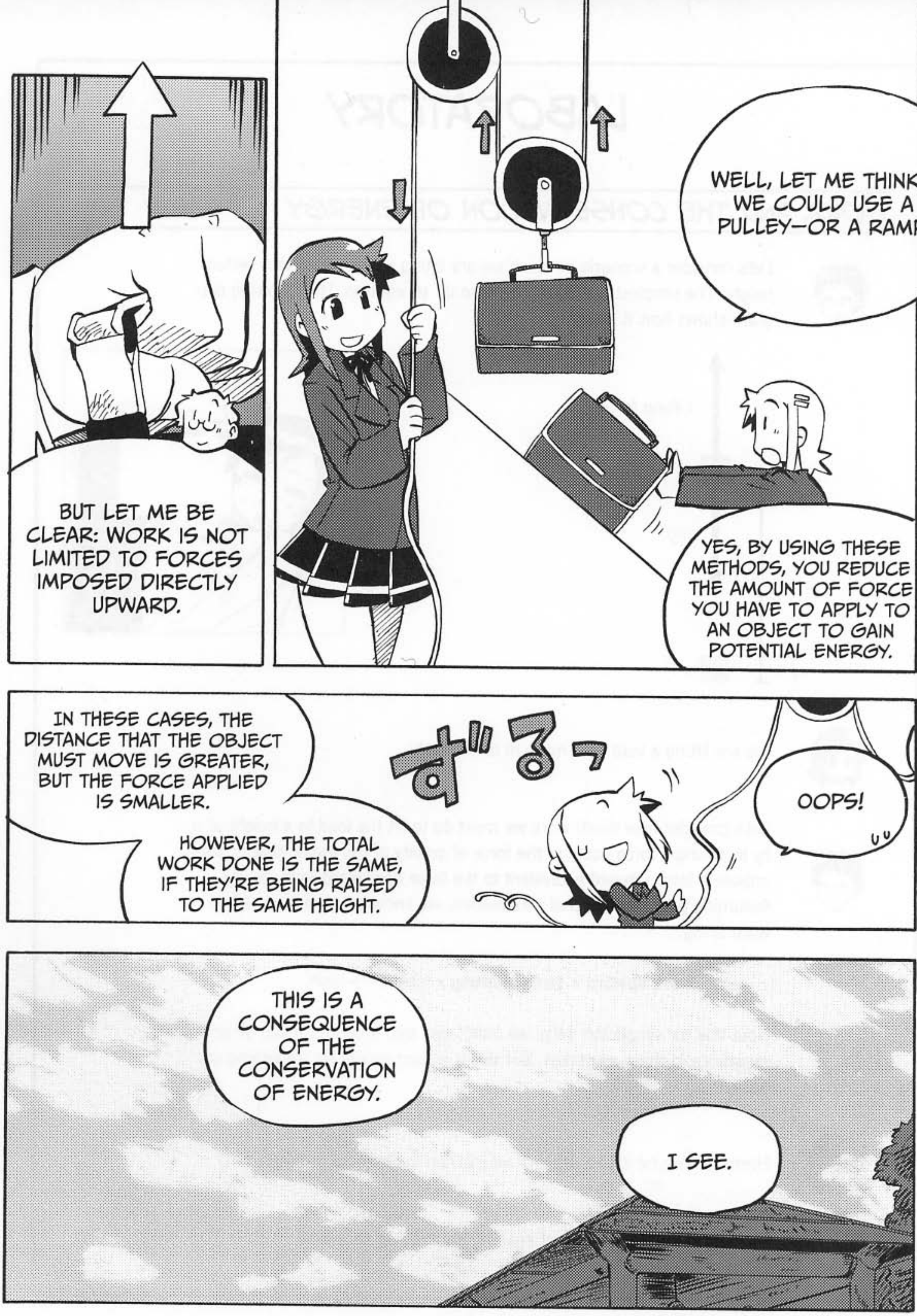

THE POTENTIAL ENERGY THAT COMES FROM HEIGHT IS CALLED *GRAVITATIONAL POTENTIAL ENERGY*
BECAUSE ITS SOURCE IS THE GRAVITY OF EARTH.
YOU MEAN THERE ARE OTHER KINDS OF POTENTIAL ENERGY?
CERTAINLY. FOR EXAMPLE, CONSIDER A RUBBER BAND OR A SPRING.
ぱっ
HE HAS SO MANY TOYS...
WHEN IT'S STRETCHED OUT, A RUBBER BAND STORES POTENTIAL ENERGY.
A RUBBER BAND OR SPRING HAS ENERGY FOR RESTORING ITSELF TO ITS ORIGINAL LENGTH. THIS KIND OF POTENTIAL ENERGY IS CALLED *ELASTIC POTENTIAL ENERGY*.
WHEN YOU RELEASE THE SLINGSHOT, THE POTENTIAL ENERGY OF THE RUBBER BAND TURNS INTO KINETIC ENERGY FOR THE SHOT.


YOU MUST LIFT AN OBJECT OR PULL THE END OF A RUBBER BAND TO GIVE AN OBJECT POTENTIAL ENERGY.
IN THE SAME WAY, YOU MUST IMPOSE A FORCE ON AN OBJECT TO CREATE KINETIC ENERGY.
SPROINNNG!
THUS, IN ORDER TO TRANSFORM ENERGY, YOU MUST IMPOSE A FORCE OVER A DISTANCE.
THIS IS REFERRED TO AS WORK.
WELL, IT DOESN'T SEEM TO HAVE ANYTHING TO DO WITH BUSINESS CASUAL.
YOU'RE RIGHT. WORK IN MECHANICS IS DEFINED PRECISELY AS FOLLOWS:
WORK = DISPLACEMENT OF AN OBJECT x THE COMPONENT OF FORCE APPLIED IN THE SAME DIRECTION
SEE?
COMPONENT OF FORCE APPLIED IN THE DIRECTION OF DISPLACEMENT
FORCE
OBJECT
DISPLACEMENT OF THE OBJECT
SIMPLY PUT, WORK IS EQUAL TO THE DISTANCE MULTIPLIED BY THE FORCE...
WELL, YES, BUT WE HAVE TO CONSIDER THE ORIENTATION OF THAT FORCE, TOO.


WHEN YOU LIFT AN OBJECT DIRECTLY UPWARD, THE WORK DONE IS EQUAL TO THE FORCE APPLIED MULTIPLIED BY THE DISTANCE LIFTED.
HOWEVER, IF WE SIMPLY HOLD AN OBJECT WITHOUT MOVING IT, WE ARE NOT DOING WORK IN THE SENSE OF MECHANICS... EVEN IF WE GET REALLY TIRED.
FORCE
MOVING
YOU DO WORK WHEN LIFTING A BAG.
BUT HOLDING THAT BAG IS *NOT* WORK.
FORCE
HOLDING
I SEE. EVEN THOUGH I GET TIRED, THAT DOESN'T MEAN I'VE DONE WORK.
RYOTA'S BAG IS HEAVY!
YOU SHOULD THINK OF WORK AS A MEANS OF INCREASING OR DECREASING AN OBJECT'S ENERGY. AFTER DOING WORK ON AN OBJECT, YOU CAN SAY THAT...
THE OBJECT MUST HAVE KINETIC OR POTENTIAL ENERGY. BUT YOU CAN'T SAY, "AN OBJECT HAS WORK." WORK IS PERFORMED ON OBJECTS BY A FORCE.
PUFF
WHEW!
OKAY, I GET IT!

WORK AND POTENTIAL ENERGY
SO, YOU CAN INCREASE POTENTIAL ENERGY BY DOING WORK.
YEAH, IF YOU DO WORK TO LIFT AN OBJECT, ITS POTENTIAL ENERGY INCREASES.
FOR EXAMPLE, LET'S CONSIDER THAT BAG AGAIN.
FORCE FROM THE HAND
x
HEIGHT THE OBJECT IS RAISED
HERE, WORK HAS BEEN DONE.
THE ORIENTATION OF THE FORCE AND THAT OF MOVING THE BAG RESULTS IN A POSITIVE VALUE FOR THE AMOUNT OF WORK.
THAT MEANS THE POTENTIAL ENERGY HAS INCREASED.

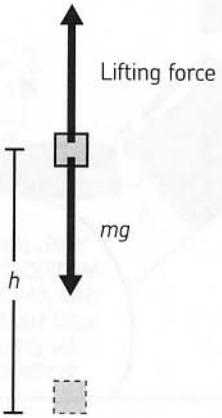

IS THE VALUE OF WORK NEGATIVE IF I LOWER THE BAG?
EXACTLY.
POTENTIAL ENERGY INCREASES
FORCE
MOVING
POSITIVE WORK
FORCE
MOVING
POTENTIAL ENERGY DECREASES
NEGATIVE WORK
WHEN YOU DECREASE THE BAG'S POTENTIAL ENERGY, THE ORIENTATION OF THE FORCE IS OPPOSITE THE DIRECTION OF MOTION, MEANING THAT NEGATIVE WORK IS DONE ON THE BAG.
SIMILARLY, WHEN YOU PULL A RUBBER BAND, YOU ARE DOING POSITIVE WORK,
STRRRRETCH
F
SINCE POTENTIAL ENERGY IS STORED.


BUT LET ME BE CLEAR: WORK IS NOT LIMITED TO FORCES IMPOSED DIRECTLY UPWARD.
WELL, LET ME THINK... WE COULD USE A PULLEY—OR A RAMP.
YES, BY USING THESE METHODS, YOU REDUCE THE AMOUNT OF FORCE YOU HAVE TO APPLY TO AN OBJECT TO GAIN POTENTIAL ENERGY.
IN THESE CASES, THE DISTANCE THAT THE OBJECT MUST MOVE IS GREATER, BUT THE FORCE APPLIED IS SMALLER.
HOWEVER, THE TOTAL WORK DONE IS THE SAME, IF THEY'RE BEING RAISED TO THE SAME HEIGHT.
ずるっ
OOPS!
THIS IS A CONSEQUENCE OF THE CONSERVATION OF ENERGY.
I SEE.

LABORATORY

WORK AND THE CONSERVATION OF ENERGY

Let's consider a scenario in which we are lifting a heavy load to a certain height. The simplest way to do this is to lift straight up. The following diagram shows how it looks.



We are lifting a load with mass m to height h.

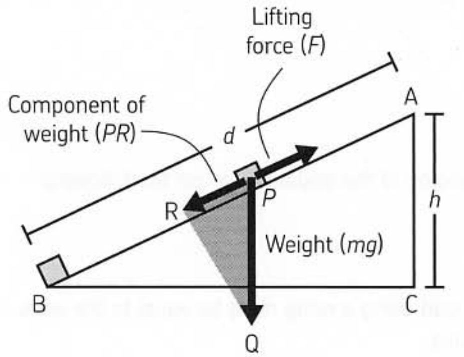
Let's consider how much work we must do to lift the load to a height of h by imposing a force equal to the force of gravity of the mass—that is, we'll impose a force upward equivalent to the force downward from gravity. Assuming g for gravitational acceleration, we know that the force downward is mg:

$$\text{work upward} = \text{force of lifting} \times \text{height } h = mgh$$

Note that for simplicity's sake, we won't take into account friction or air resistance in these examples. But this is a hard way to lift something so heavy!

Hmm . . . maybe it'd be easier if we pushed the load up a ramp.

Yes, let's consider the case of pushing the load up an incline.



Look at this diagram. The magnitude of the force needed to push the load up this ramp (F) is equal to the component of the force of gravity parallel to the ramp (PR). So, if the ramp has a length of d, the work required to move the load to height h can be represented as:

$$\text{work} = Fd$$

Now, you know intuitively that F is smaller than mg, and d is larger than h.

That makes sense. Is that why it takes the same amount of work to push the load up a ramp as it does to lift the load straight up?

Yes, indeed. Now let's show why this works, mathematically. $\triangle ABC$ represents the ramp in the figure, and $\triangle PQR$ represents the composition of the force mg. These two triangles are similar—this means that $\angle CAB = \angle RPQ$. This also means that the proportion of their corresponding sides must be the same, as well. Thus, the following must be true:

$$\frac{AB}{AC} = \frac{PQ}{PR}$$

Let's make this a little less abstract. The line segment AB equals d (length of ramp) and AC equals h (height). Similarly, the line segment PQ equals mg (the force downward, due to gravity), while PR equals F (the force applied to offset a portion of that force).

That means:

$$\frac{d}{h} = \frac{mg}{F}$$

Look, with just a little rearranging of this equation we get the following:

$$Fd = mgh$$

Therefore, the work to lift a load using a ramp must be equal to the work to lift that load straight upward.

Also, please note that our results are the same, regardless of the angle of the ramp. Given the conservation of energy, regardless of the lifting route, the work done for lifting an object with mass m to height h is equal to the following:

$$\text{force required to balance gravity} \times \text{height} = mgh$$

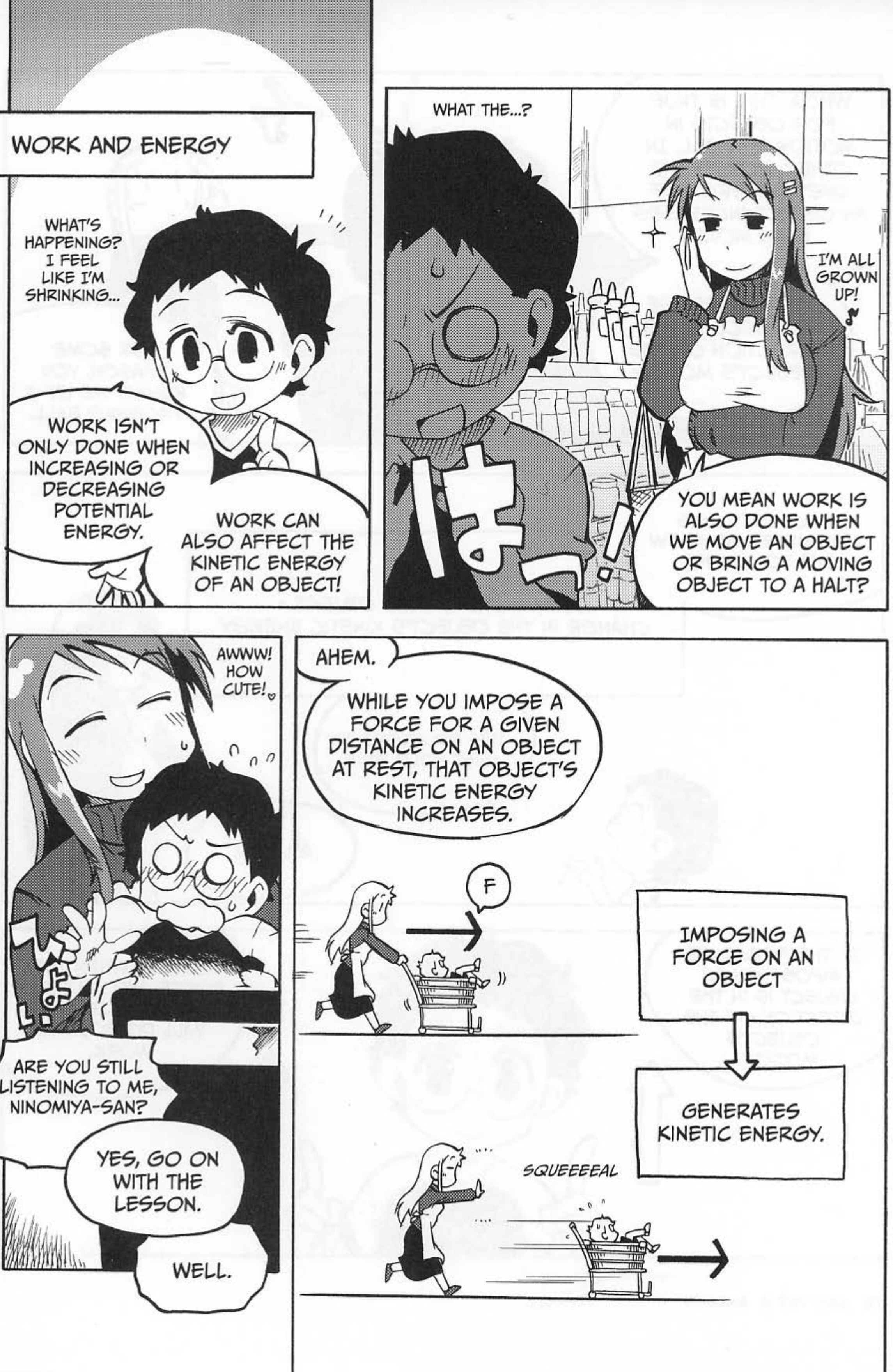
So, whatever method you use to lift something, the amount of work you do is the same.

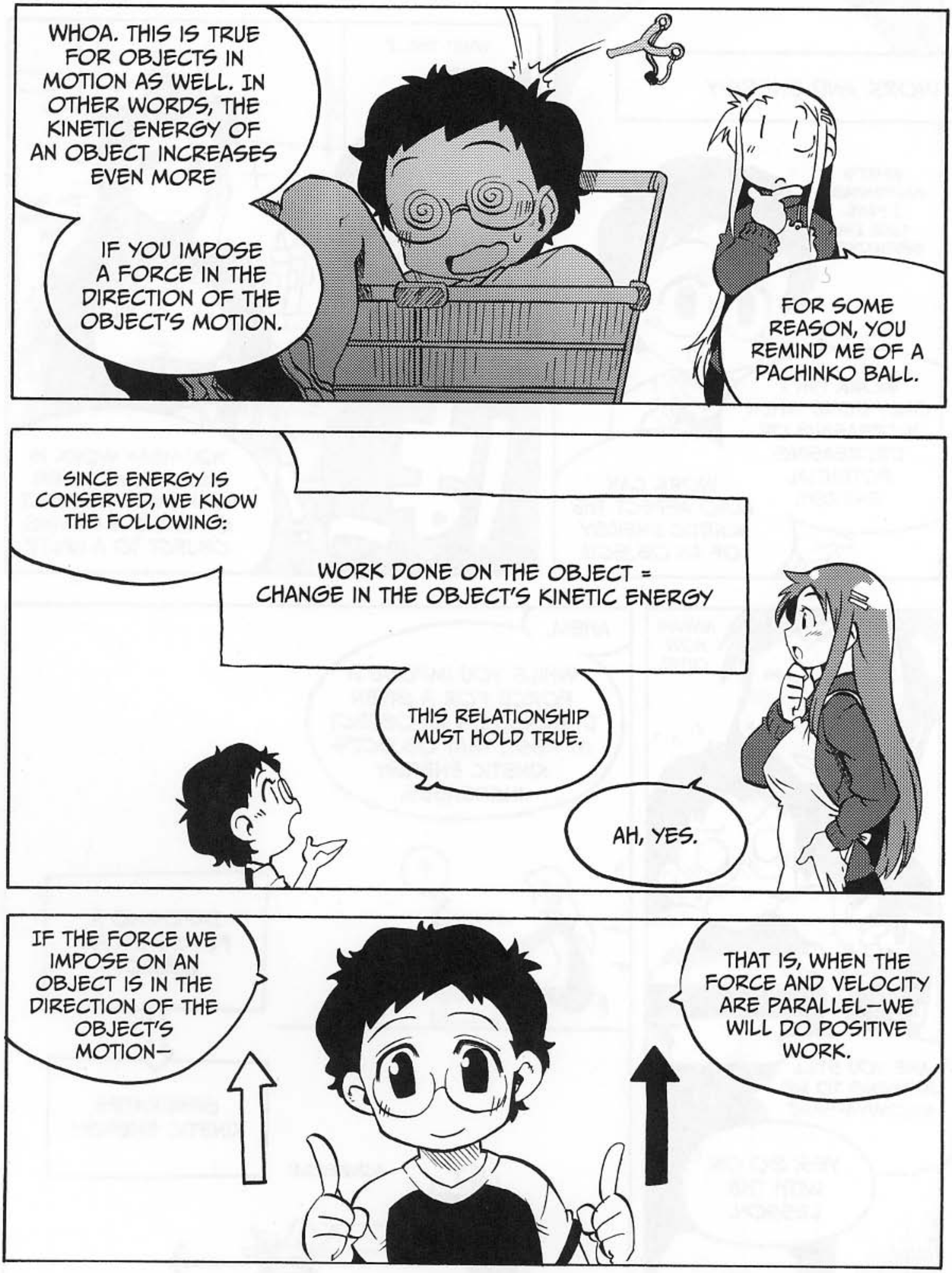
To put it another way, your work increases the potential energy of the load by mgh.

And I bet it works for negative work, too. That is, you'd see a decrease in potential energy of mgh if you lower an object by mgh.

Yep, that's right.

WORK AND ENERGY



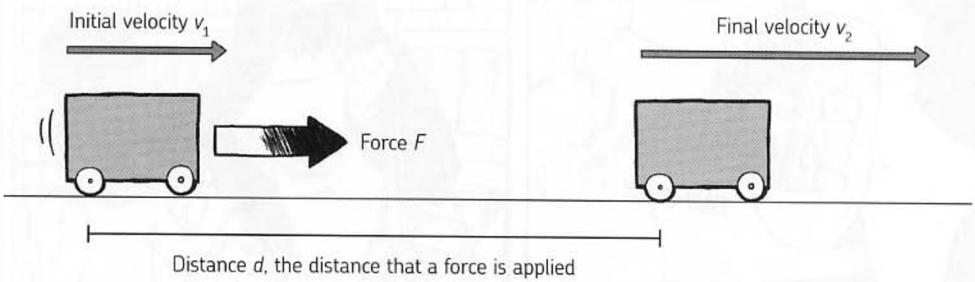

WHOA. THIS IS TRUE FOR OBJECTS IN MOTION AS WELL. IN OTHER WORDS, THE KINETIC ENERGY OF AN OBJECT INCREASES EVEN MORE
IF YOU IMPOSE A FORCE IN THE DIRECTION OF THE OBJECT'S MOTION.
FOR SOME REASON, YOU REMIND ME OF A PACHINKO BALL.
SINCE ENERGY IS CONSERVED, WE KNOW THE FOLLOWING:
WORK DONE ON THE OBJECT = CHANGE IN THE OBJECT'S KINETIC ENERGY
THIS RELATIONSHIP MUST HOLD TRUE.
AH, YES.
IF THE FORCE WE IMPOSE ON AN OBJECT IS IN THE DIRECTION OF THE OBJECT'S MOTION—
THAT IS, WHEN THE FORCE AND VELOCITY ARE PARALLEL—WE WILL DO POSITIVE WORK.


WE KNOW THAT THERE'S BEEN A POSITIVE CHANGE IN KINETIC ENERGY—THAT IS, THE OBJECT IS SPEEDING UP.
I WANT SOME POCKY, MOMMY!
THE KINETIC KID
TAH-RAH
LIKEWISE, YOU CAN STOP AN OBJECT IN MOTION BY IMPOSING A FORCE IN A DIRECTION OPPOSITE TO ITS VELOCITY.
REDUCING ITS KINETIC ENERGY, I SUPPOSE.
AT THIS TIME, THE ORIENTATION OF VELOCITY AND THE FORCE ARE OPPOSED TO EACH OTHER, SO THE VALUE OF WORK WILL BE NEGATIVE.
THEREFORE, THE CHANGE IN KINETIC ENERGY ALSO TAKES A NEGATIVE VALUE—IT DECREASES.
THAT WAS BIZARRE. I'M GLAD I'M BACK TO MY OLD SELF AGAIN.
WHEW!

LABORATORY

THE RELATIONSHIP BETWEEN WORK AND KINETIC ENERGY

Let's examine how we can derive an equation that expresses the relationship between work and kinetic energy. Suppose we continue to impose force F on a cart in motion, in a direction parallel to that cart's velocity. That cart has mass m and starts with an initial, uniform velocity of v.



That means an additional force is imposed on the object in motion.

At this time, the following is true:

$$\text{work done on the object} = Fd$$

Also, since we've represented the final velocity as v_2, we can represent the change in the object's kinetic energy as the following:

$$\text{change in kinetic energy} = \tfrac{1}{2}mv_2^{\,2} - \tfrac{1}{2}mv_1^{\,2}$$

And since we already know that the change in kinetic energy is equal to the work done on the object, we can express the following relationship:

$$\tfrac{1}{2}mv_2^{\,2} - \tfrac{1}{2}mv_1^{\,2} = Fd$$

Aha.

We can also derive this equation another way. Since F is defined as constant, the cart is experiencing uniform acceleration. Therefore, if we represent the cart's acceleration with a, we know that the following must be true:

$$v_2^2 - v_1^2 = 2ad$$

(Why is this so? See expression ❸ on page 85.) To get closer to our original expression, we'll substitute using Newton's second law:

$$F = ma, \text{ or rearranged just a little, } a = \frac{F}{m}$$

And we'll get the following:

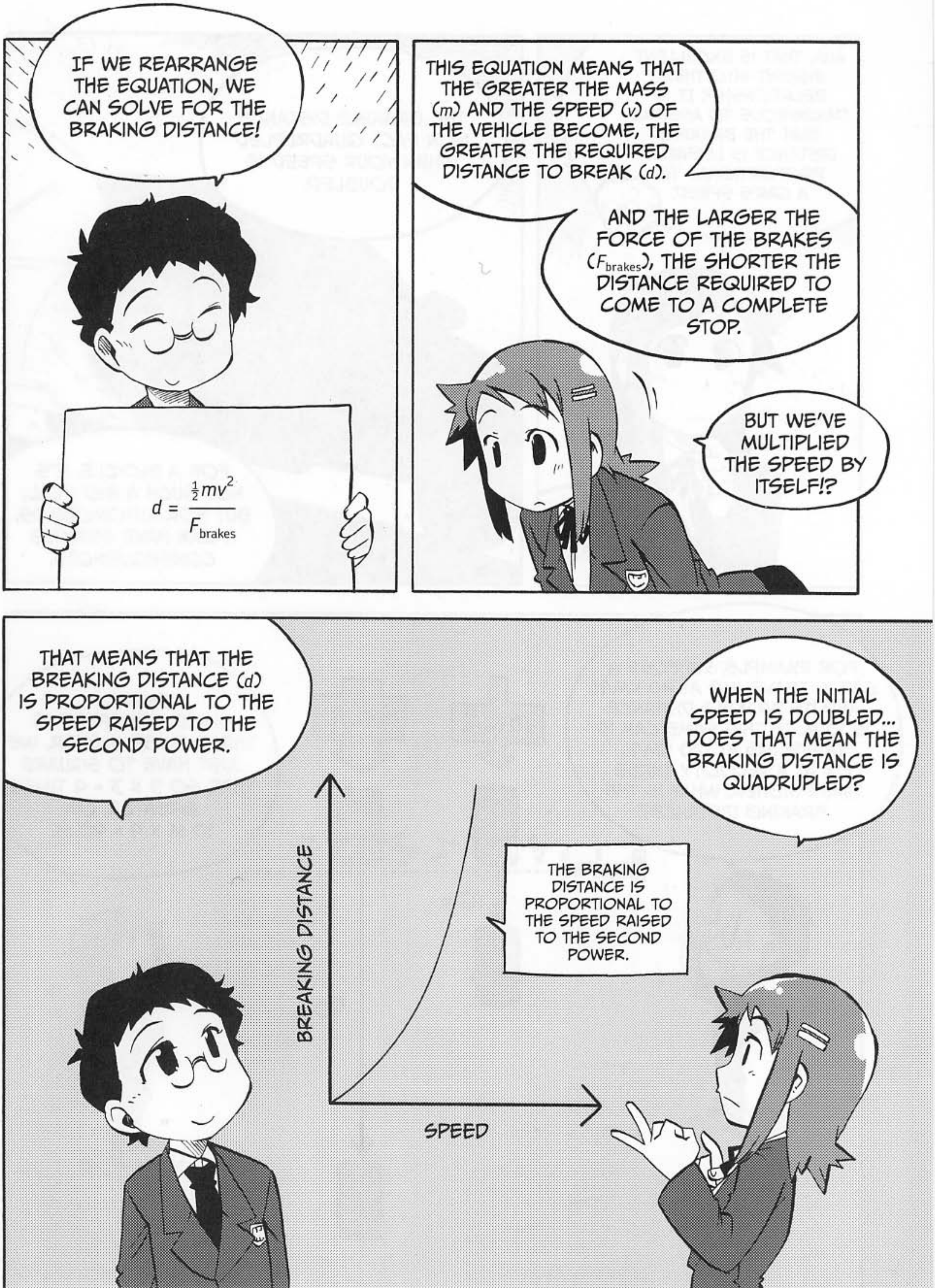
$$v_2^2 - v_1^2 = \frac{2Fd}{m}$$

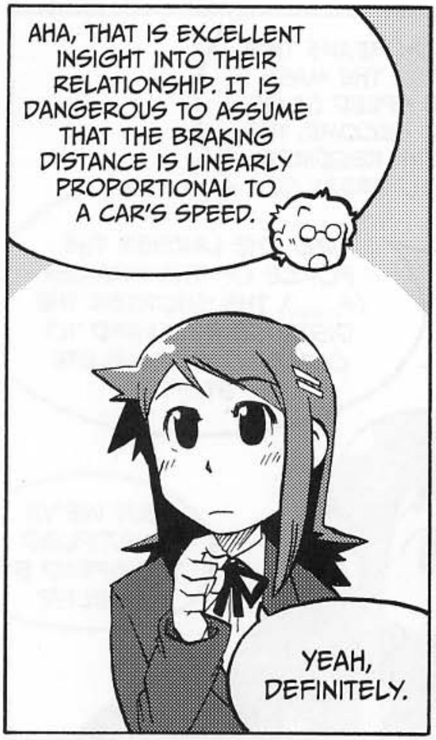
Then if you simply multiply both sides by $\frac{1}{2}$, you're there!

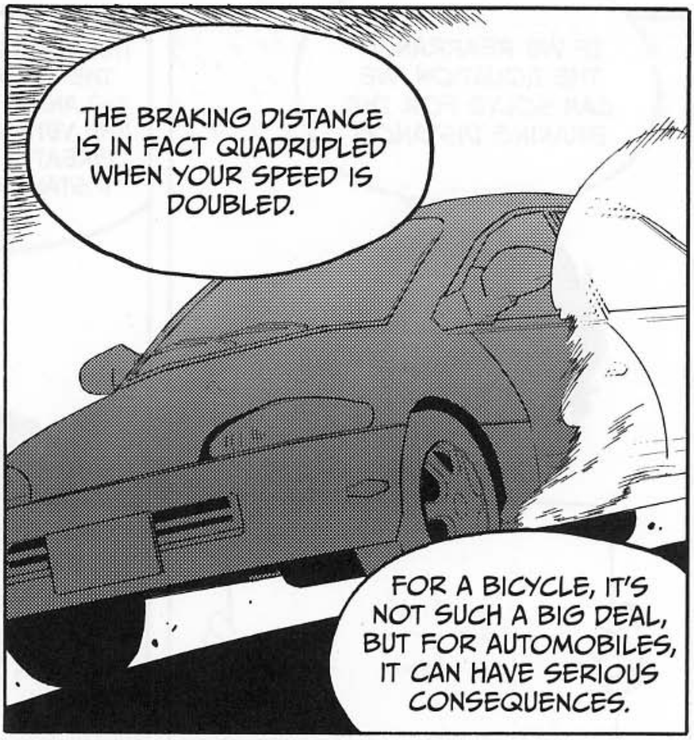
$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = Fd$$

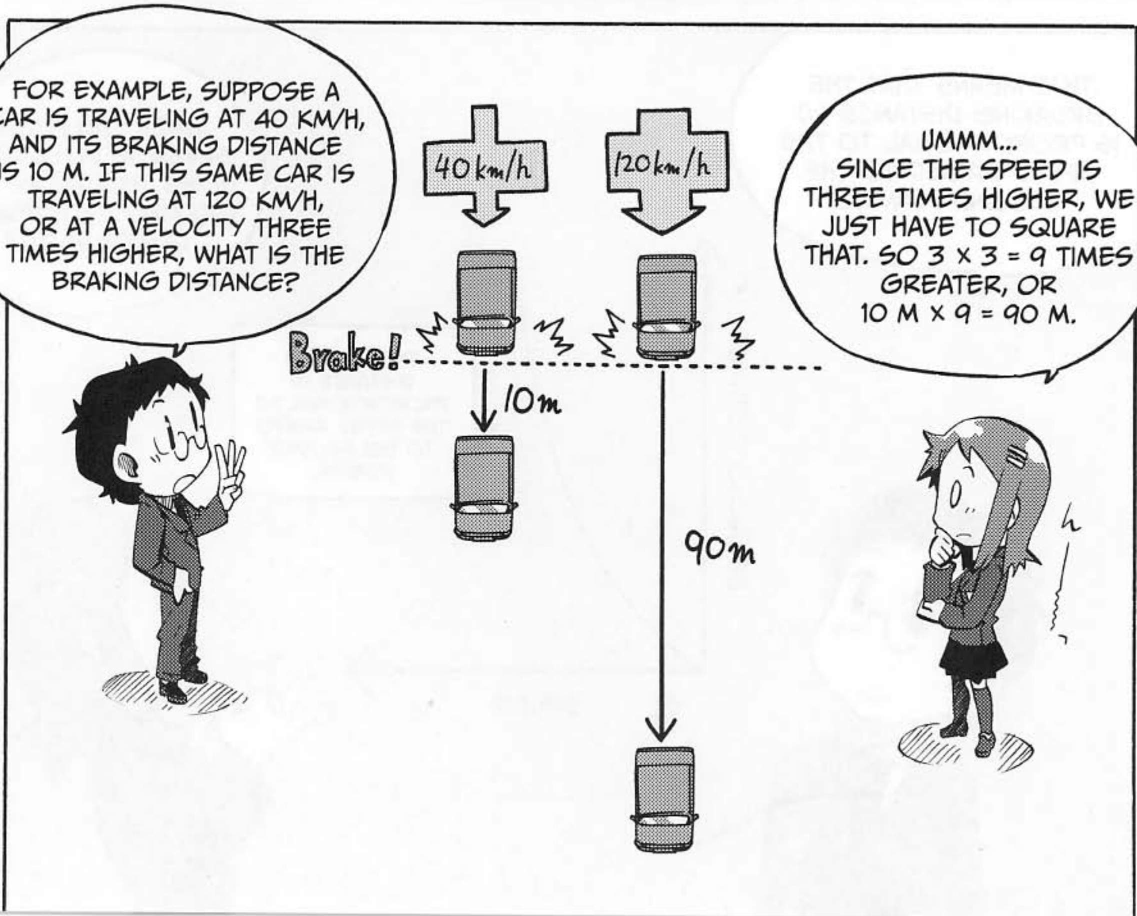
I can get it right if I calculate very carefully.

BRAKING DISTANCE AND SPEED
USING WHAT WE KNOW ABOUT THE RELATIONSHIP BETWEEN KINETIC ENERGY AND WORK, LET'S CONSIDER A CAR'S BRAKING DISTANCE.
WHAT DO YOU MEAN, EXACTLY?
WELL, I GUESS IT'S NOT JUST FOR CARS. IT'S THE DISTANCE THAT ANY OBJECT IN MOTION REQUIRES TO STOP,
GIVEN A CERTAIN FORCE IN THE OPPOSITE DIRECTION.
GIVEN THAT WE KNOW A CHANGE IN KINETIC ENERGY IS EQUAL TO THE WORK PERFORMED, WE KNOW THAT THE FOLLOWING MUST BE TRUE OF BRINGING AN OBJECT IN MOTION TO REST:
½ MASS × SPEED² = FORCE OF THE BRAKES × DISTANCE THE BRAKES ARE APPLIED
$\frac{1}{2}mv^2 = F_{brakes} \times d$


IF WE REARRANGE THE EQUATION, WE CAN SOLVE FOR THE BRAKING DISTANCE!
$d = \frac{\frac{1}{2}mv^2}{F_{brakes}}$
THIS EQUATION MEANS THAT THE GREATER THE MASS (m) AND THE SPEED (v) OF THE VEHICLE BECOME, THE GREATER THE REQUIRED DISTANCE TO BREAK (d).
AND THE LARGER THE FORCE OF THE BRAKES (F_{brakes}), THE SHORTER THE DISTANCE REQUIRED TO COME TO A COMPLETE STOP.
BUT WE'VE MULTIPLIED THE SPEED BY ITSELF!?
THAT MEANS THAT THE BREAKING DISTANCE (d) IS PROPORTIONAL TO THE SPEED RAISED TO THE SECOND POWER.
BREAKING DISTANCE
SPEED
THE BRAKING DISTANCE IS PROPORTIONAL TO THE SPEED RAISED TO THE SECOND POWER.
WHEN THE INITIAL SPEED IS DOUBLED... DOES THAT MEAN THE BRAKING DISTANCE IS QUADRUPLED?


AHA, THAT IS EXCELLENT INSIGHT INTO THEIR RELATIONSHIP. IT IS DANGEROUS TO ASSUME THAT THE BRAKING DISTANCE IS LINEARLY PROPORTIONAL TO A CAR'S SPEED.
YEAH, DEFINITELY.

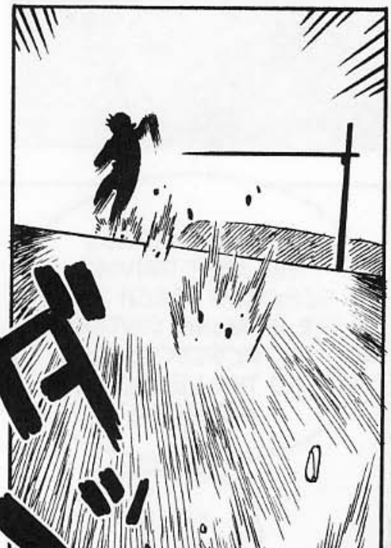

THE BRAKING DISTANCE IS IN FACT QUADRUPLED WHEN YOUR SPEED IS DOUBLED.
FOR A BICYCLE, IT'S NOT SUCH A BIG DEAL, BUT FOR AUTOMOBILES, IT CAN HAVE SERIOUS CONSEQUENCES.

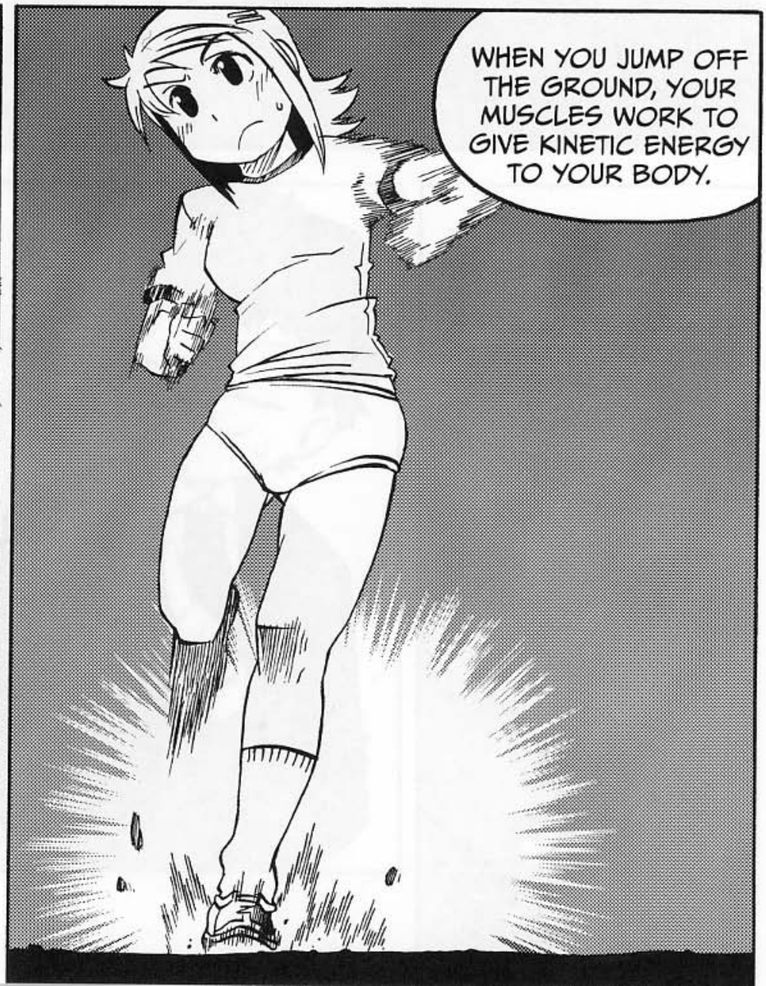

FOR EXAMPLE, SUPPOSE A CAR IS TRAVELING AT 40 KM/H, AND ITS BRAKING DISTANCE IS 10 M. IF THIS SAME CAR IS TRAVELING AT 120 KM/H, OR AT A VELOCITY THREE TIMES HIGHER, WHAT IS THE BRAKING DISTANCE?
40 km/h
120 km/h
Brake!
10m
90m
UMMM... SINCE THE SPEED IS THREE TIMES HIGHER, WE JUST HAVE TO SQUARE THAT. SO 3 × 3 = 9 TIMES GREATER, OR 10 M × 9 = 90 M.

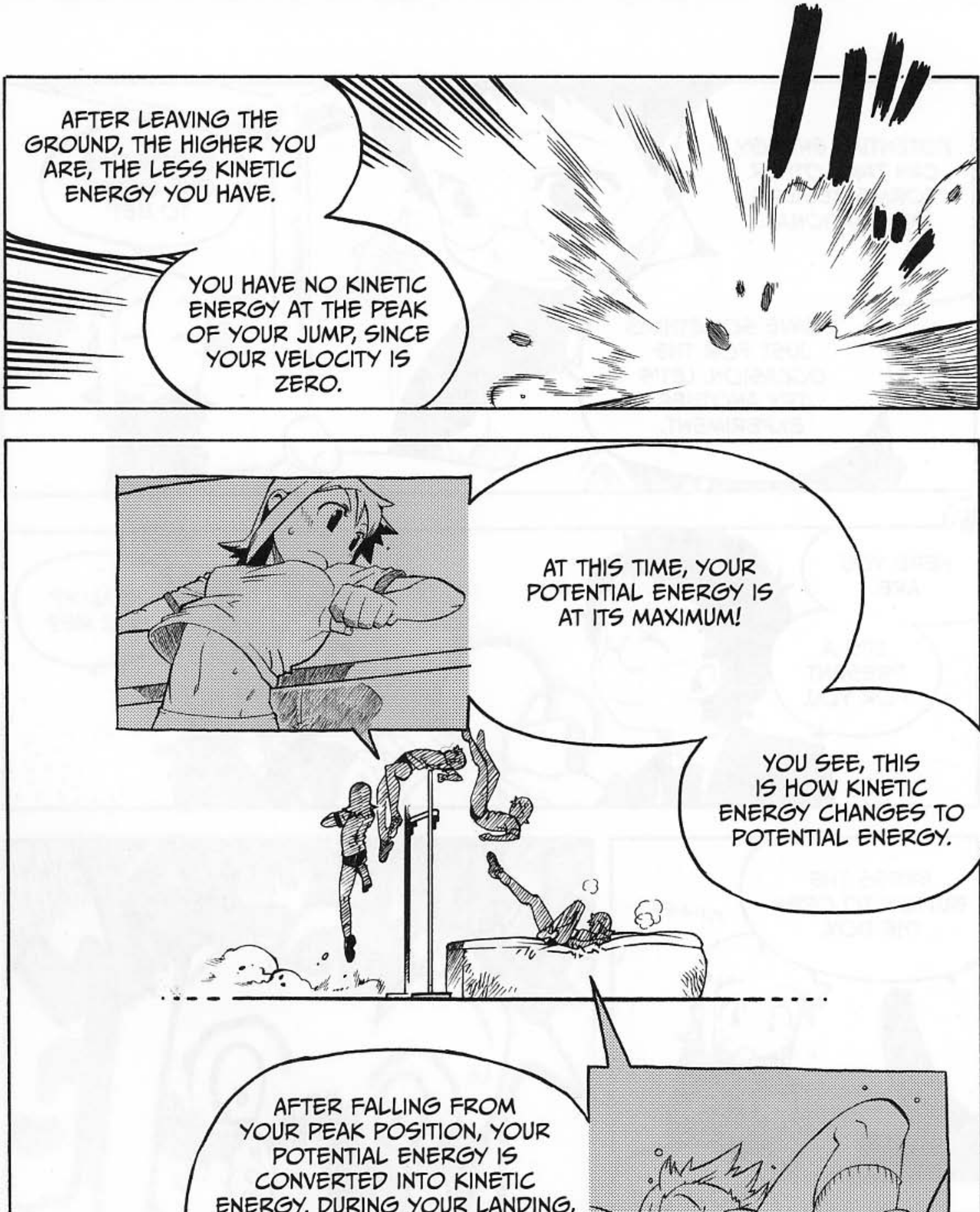

SAFE DRIVERS WOULD DO WELL TO KEEP THIS PRINCIPLE IN MIND.
VRRRRRRROOM
YOU'D BETTER START BRAKING NOW!
DON'T WORRY, I STILL HAVE TIME!
IF WE MISTAKENLY ASSUMED THAT OUR STOPPING DISTANCE WAS LINEARLY RELATED TO SPEED, WE'D THINK IT WAS ONLY 30 M. WE'D BE OFF BY 60 M!
DESPITE THIS DRIVER'S CONFIDENCE, A GRISLY ACCIDENT IS VERY POSSIBLE, SINCE THE BRAKING DISTANCE IS SO LARGE.
I HEAR THAT ALL THE BEST DRIVING SCHOOLS TEACH THAT THE BRAKING DISTANCE IS PROPORTIONAL TO THE VELOCITY SQUARED.
OH, I'M SURE.
YIKES!

THE CONSERVATION OF MECHANICAL ENERGY

TRANSFORMING ENERGY

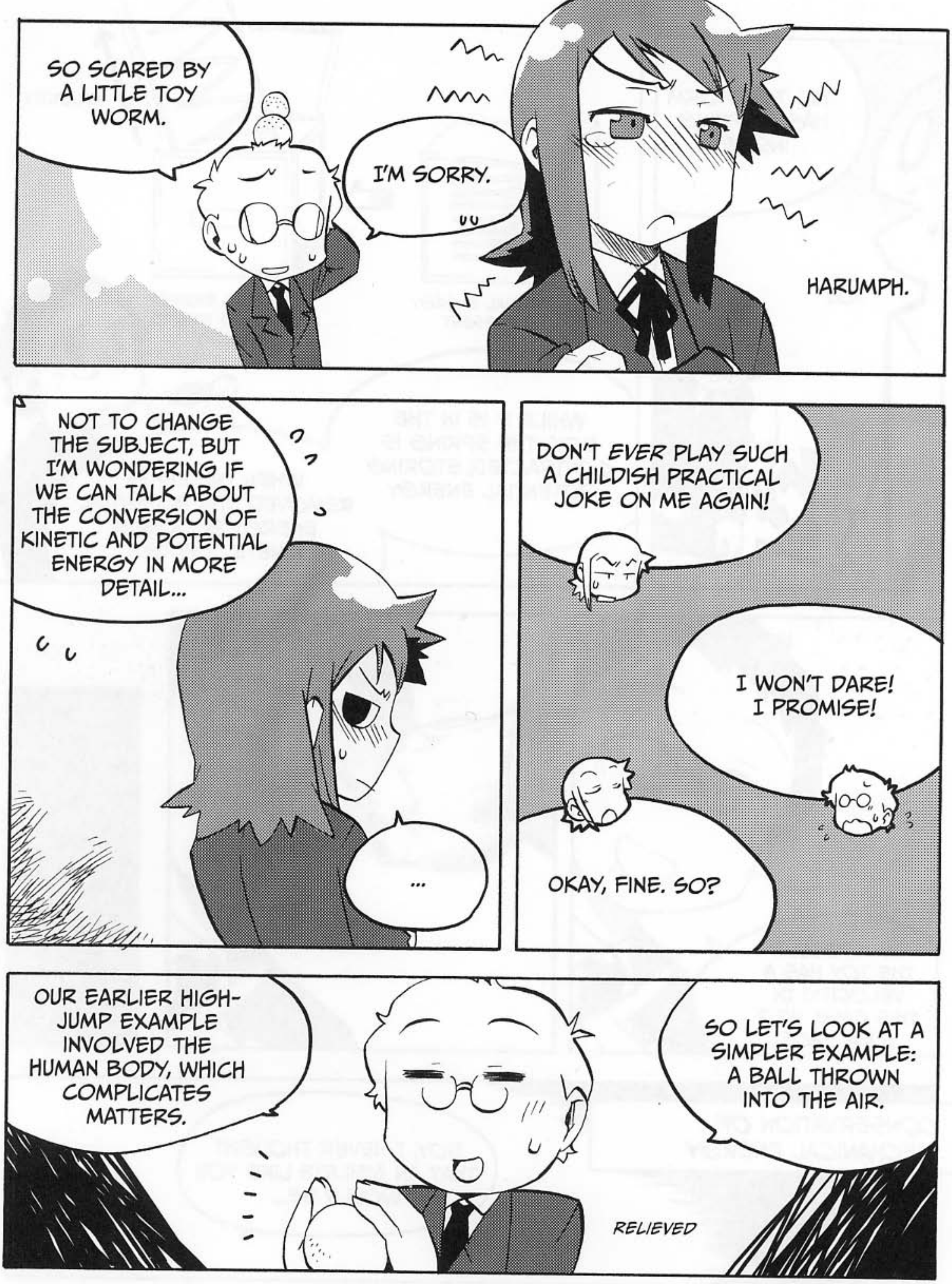


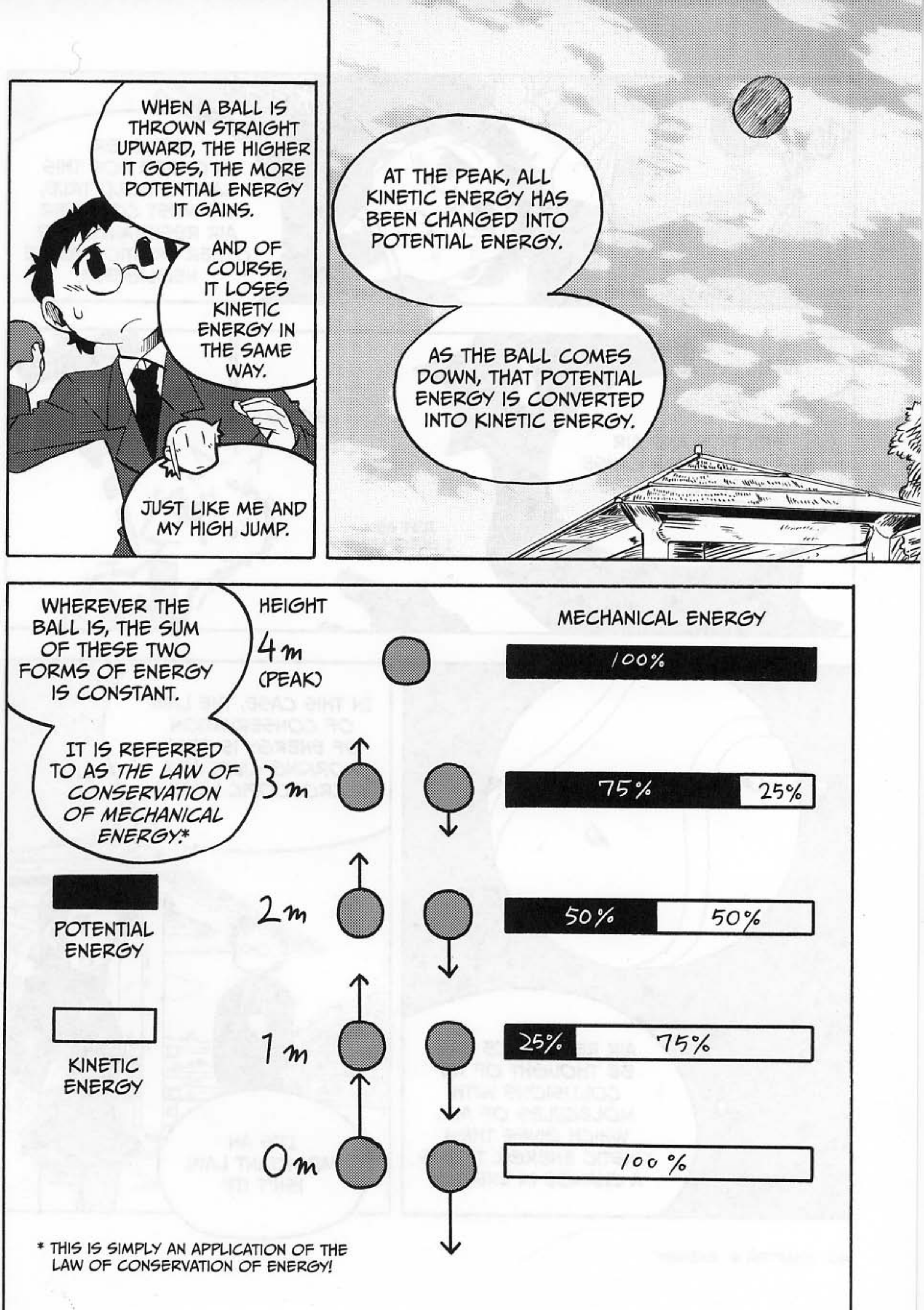



AFTER LEAVING THE GROUND, THE HIGHER YOU ARE, THE LESS KINETIC ENERGY YOU HAVE.
YOU HAVE NO KINETIC ENERGY AT THE PEAK OF YOUR JUMP, SINCE YOUR VELOCITY IS ZERO.
AT THIS TIME, YOUR POTENTIAL ENERGY IS AT ITS MAXIMUM!
YOU SEE, THIS IS HOW KINETIC ENERGY CHANGES TO POTENTIAL ENERGY.
AFTER FALLING FROM YOUR PEAK POSITION, YOUR POTENTIAL ENERGY IS CONVERTED INTO KINETIC ENERGY. DURING YOUR LANDING, THE MAT DOES NEGATIVE WORK ON YOUR BODY, AS YOUR KINETIC ENERGY DECREASES.

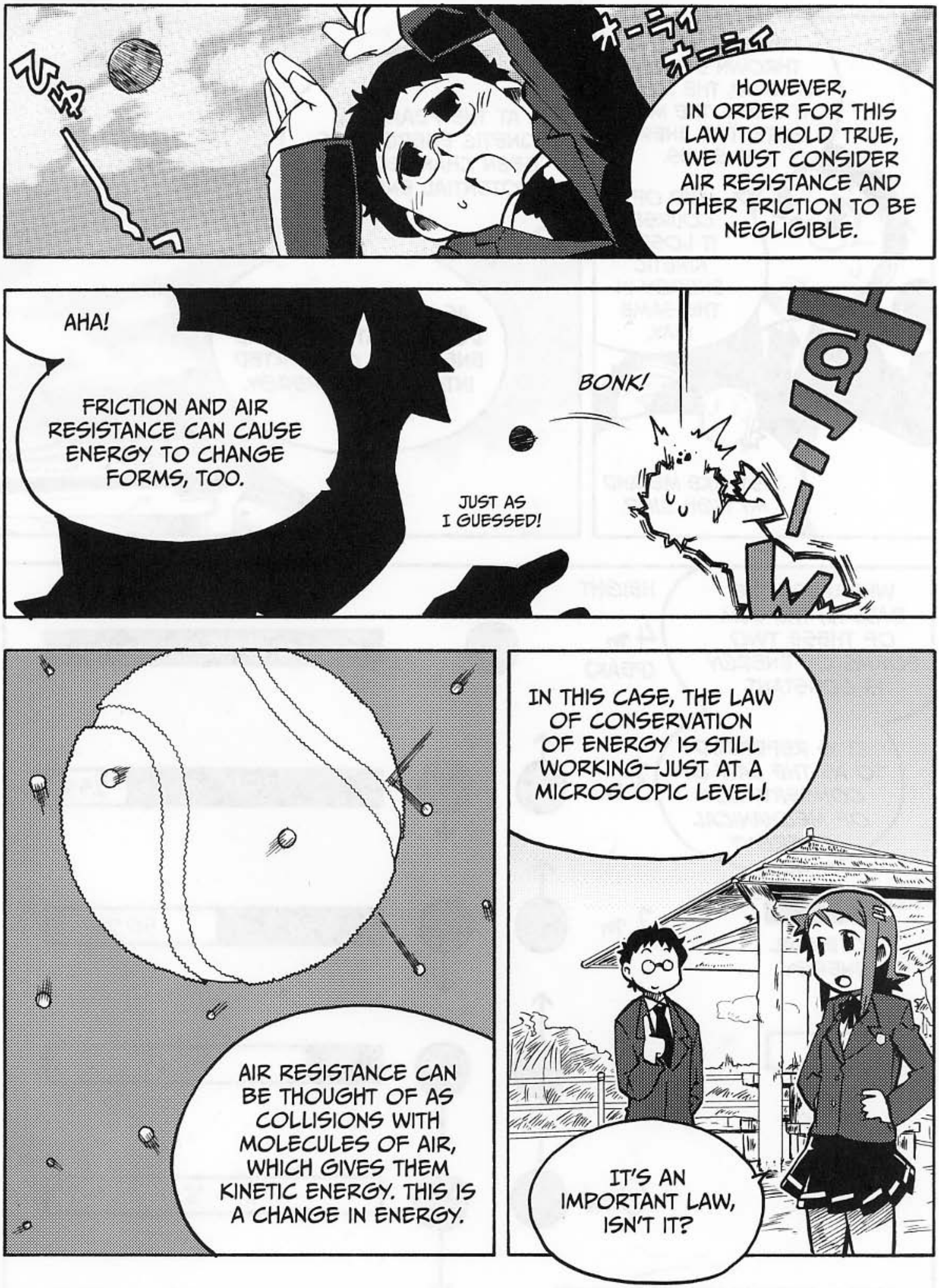

POTENTIAL ENERGY CAN TAKE OTHER FORMS BESIDES GRAVITATIONAL.
WHAT'S THIS ONE GOING TO BE?
I HAVE SOMETHING JUST FOR THE OCCASION. LET'S TRY ANOTHER EXPERIMENT.
HERE YOU ARE.
IT'S A PRESENT FOR YOU.
REALLY? FOR ME?
PRESS THE BUTTON TO OPEN THE BOX.
PIT-A-PAT
WOW, YOU DIDN'T HAVE TO...
CLICK
POP


THIS TOY WORM HAS A SPRING INSIDE.
ACK!
POTENTIAL ENERGY IS PRESENT
POTENTIAL ENERGY BECOMES KINETIC ENERGY
VELOCITY
WHILE IT IS IN THE BOX, THE SPRING IS CONTRACTED, STORING POTENTIAL ENERGY.
WHEN THE LID IS REMOVED, THE POTENTIAL ENERGY BECOMES KINETIC ENERGY.
AS A RESULT THE TOY HAS A VELOCITY! IN THIS CASE, INTO YOUR FACE.
CONSERVATION OF MECHANICAL ENERGY
BOY, I NEVER THOUGHT THAT AN ATHLETE LIKE YOU WOULD BE...


SO SCARED BY A LITTLE TOY WORM.
I'M SORRY.
HARUMPH.
NOT TO CHANGE THE SUBJECT, BUT I'M WONDERING IF WE CAN TALK ABOUT THE CONVERSION OF KINETIC AND POTENTIAL ENERGY IN MORE DETAIL...
...
DON'T *EVER* PLAY SUCH A CHILDISH PRACTICAL JOKE ON ME AGAIN!
I WON'T DARE! I PROMISE!
OKAY, FINE. SO?
OUR EARLIER HIGH-JUMP EXAMPLE INVOLVED THE HUMAN BODY, WHICH COMPLICATES MATTERS.
SO LET'S LOOK AT A SIMPLER EXAMPLE: A BALL THROWN INTO THE AIR.
RELIEVED



* THIS IS SIMPLY AN APPLICATION OF THE LAW OF CONSERVATION OF ENERGY!


HOWEVER, IN ORDER FOR THIS LAW TO HOLD TRUE, WE MUST CONSIDER AIR RESISTANCE AND OTHER FRICTION TO BE NEGLIGIBLE.
AHA!
FRICTION AND AIR RESISTANCE CAN CAUSE ENERGY TO CHANGE FORMS, TOO.
BONK!
JUST AS I GUESSED!
AIR RESISTANCE CAN BE THOUGHT OF AS COLLISIONS WITH MOLECULES OF AIR, WHICH GIVES THEM KINETIC ENERGY. THIS IS A CHANGE IN ENERGY.
IN THIS CASE, THE LAW OF CONSERVATION OF ENERGY IS STILL WORKING—JUST AT A MICROSCOPIC LEVEL!
IT'S AN IMPORTANT LAW, ISN'T IT?

LABORATORY

THE LAW OF CONSERVATION OF MECHANICAL ENERGY IN ACTION

Let's prove that the law of conservation of mechanical energy applies when throwing a ball straight upward.

First, we know that the equation for a change in kinetic energy and work is as follows:

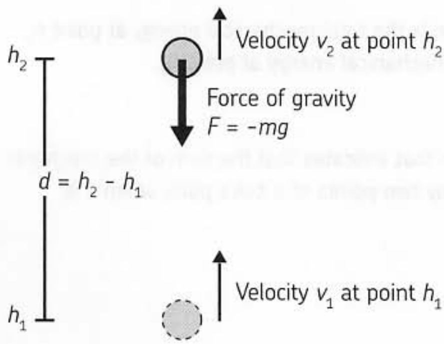
$$❶ \quad \tfrac{1}{2}mv_2^{\,2} - \tfrac{1}{2}mv_1^{\,2} = Fd$$

That is:

the change in KE = work

Yes, we confirmed that earlier.

In this case, the work Fd represents the work done by gravity. Assume that the ball starts at height h_1 with velocity v_1. After traveling distance d, it is at height h_2, and its velocity has diminished to v_2. The distance d can be thought of as the change in height—or $h_2 - h_1$.



Yeah, so what's the big deal? Are you trying to show that the force of gravity is doing negative work on the ball?

Exactly. The force of gravity is acting against the direction of the velocity. So it's expressed as:

$$F = -mg$$

That means that the work done by the ball (force × distance) is equal to:

$$Fd = -mg(h_2 - h_1)$$

Substituting values from the first equation ❶, we get the following:

$$\tfrac{1}{2}mv_2{}^2 - \tfrac{1}{2}mv_1{}^2 = -mg(h_2 - h_1)$$

Now, let's rewrite it a few times, first expanding the terms on the left side:

$$\tfrac{1}{2}mv_2{}^2 - \tfrac{1}{2}mv_1{}^2 = mgh_1 - mgh_2$$

Then, make a little switcheroo, and we have something that should be familiar:

$$\tfrac{1}{2}mv_2{}^2 + mgh_2 = \tfrac{1}{2}mv_1{}^2 + mgh_1$$

Yes, it is. It's showing that the sum of the kinetic energy and potential energy at both h_1 and h_2 must be the same.

Yes, that's it exactly.

So the left side of this equation is the total mechanical energy at point h_2, and the right side is the total mechanical energy at point h_1.

Yes, we've derived an equation that indicates that the sum of the mechanical energy must be equal at any two points of a ball's path, when it is thrown directly into the air.

Yes, I see that.

Now, let's use this equation to calculate something a bit different—the velocity (v_1) at which you need to throw a ball to reach a certain *maximum* height (h_2). Since the ball's velocity reaches zero at the peak, we know it has no kinetic energy at that time.

And for simplicity's sake, let's set h_1 equal to 0—that is, we'll measure h_2 from the ball's launching point. That is, h_2 will equal d, the distance the ball travels.

This means that the kinetic energy the ball has at its launching point must equal the potential energy it has at its height.

Therefore, the following is true:

$$PE_2 = KE_1$$

$$mgd = \tfrac{1}{2}mv_1^{\ 2}$$

Wait, I think I see something interesting here—mass appears on both sides of this equation. That means that the mass does not affect the relationship!

You're right! Let's solve for the initial velocity v_1:

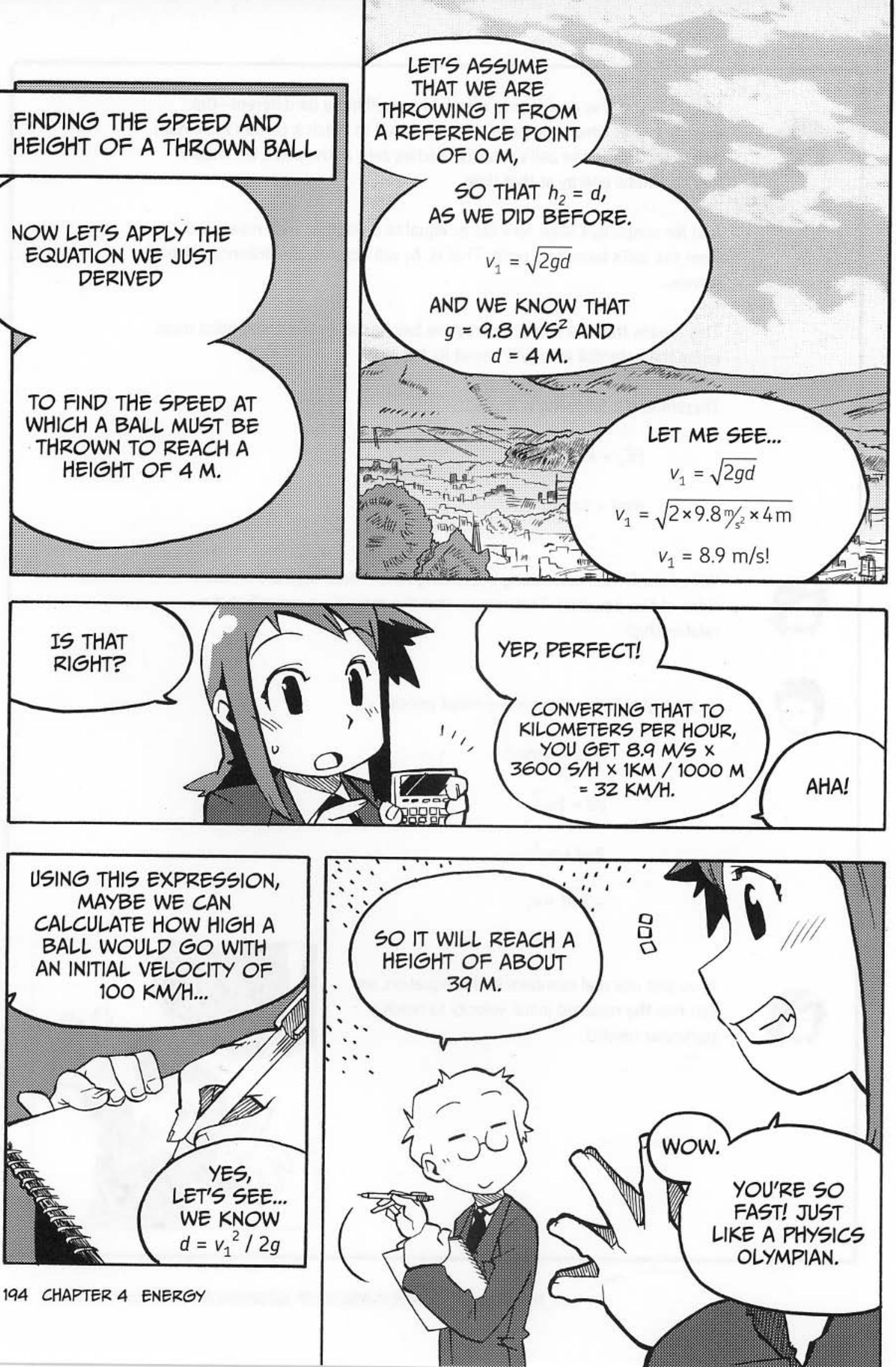
$$mgd = \tfrac{1}{2}mv_1^{\ 2}$$

$$gd = \tfrac{1}{2}v_1^{\ 2}$$

$$2gd = v_1^{\ 2}$$

$$\sqrt{2gd} = v_1$$

If we just use real numbers in this equation, we can find the required initial velocity to reach a particular height!

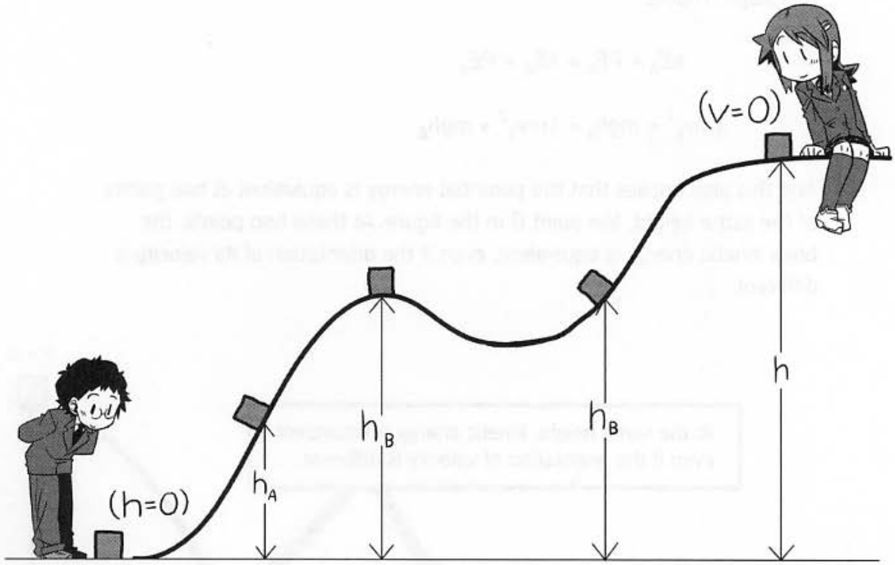

FINDING THE SPEED AND HEIGHT OF A THROWN BALL
NOW LET'S APPLY THE EQUATION WE JUST DERIVED
TO FIND THE SPEED AT WHICH A BALL MUST BE THROWN TO REACH A HEIGHT OF 4 M.
LET'S ASSUME THAT WE ARE THROWING IT FROM A REFERENCE POINT OF 0 M,
SO THAT $h_2 = d$, AS WE DID BEFORE.
$v_1 = \sqrt{2gd}$
AND WE KNOW THAT g = 9.8 M/S² AND d = 4 M.
LET ME SEE...
$v_1 = \sqrt{2gd}$
$v_1 = \sqrt{2 \times 9.8\,\mathrm{m/s^2} \times 4\,\mathrm{m}}$
v_1 = 8.9 m/s!
IS THAT RIGHT?
YEP, PERFECT!
CONVERTING THAT TO KILOMETERS PER HOUR, YOU GET 8.9 M/S × 3600 S/H × 1KM / 1000 M = 32 KM/H.
AHA!
USING THIS EXPRESSION, MAYBE WE CAN CALCULATE HOW HIGH A BALL WOULD GO WITH AN INITIAL VELOCITY OF 100 KM/H...
YES, LET'S SEE... WE KNOW $d = v_1^2 / 2g$
SO IT WILL REACH A HEIGHT OF ABOUT 39 M.
WOW.
YOU'RE SO FAST! JUST LIKE A PHYSICS OLYMPIAN.

LABORATORY

CONSERVATION OF MECHANICAL ENERGY ON A SLOPE

The law of conservation of mechanical energy holds true, even when you're not talking about balls in the air, right? Wouldn't it work for lots of other situations, too, like an object on a slope?

Well, let's examine a case where you slide a box from height h to height 0. On the way down, let's assume that the box attains velocity v_A at height h_A, velocity v_B at height h_B, and so on.



Since $v = 0$ at the highest height, the initial potential energy the box has is equal to all its mechanical energy. But we also know that the potential energy at point h is mgh, so we could express that as:

$$PE_h = mgh$$

Now, how can you express the kinetic energy (KE_0) the box has at point 0?

We already know that kinetic energy is equal to this:

$$KE_0 = \tfrac{1}{2}mv^2$$

Exactly! And we know that kinetic energy at $h = 0$ must equal the potential energy at point h:

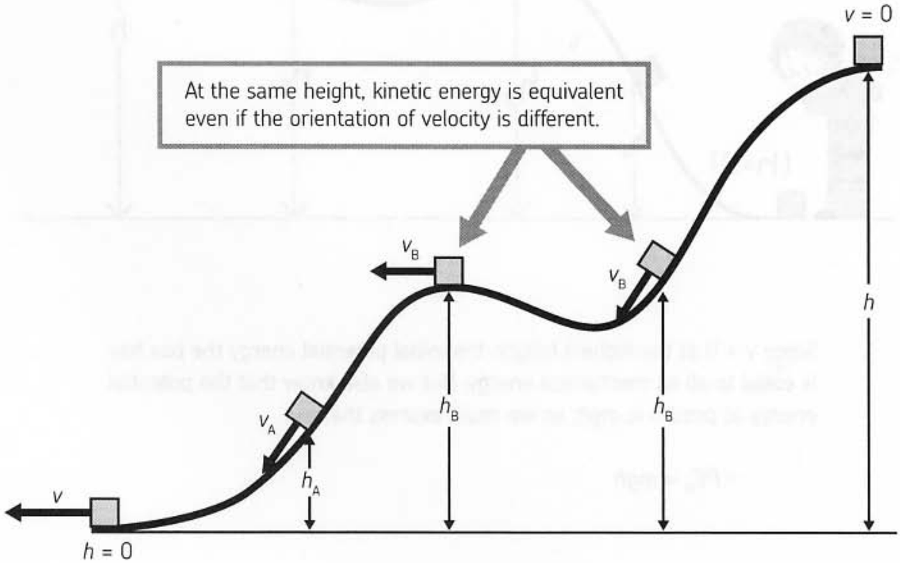
$$PE_h = KE_0$$

But furthermore, due to the conservation of energy, we know that the sum of the mechanical energy must stay the same at all intermediate points on this slope. That is:

$$KE_A + PE_A = KE_B + PE_A$$

$$\tfrac{1}{2}mv_A^{\,2} + mgh_A = \tfrac{1}{2}mv_B^{\,2} + mgh_B$$

And this also implies that the potential energy is equivalent at two points of the same height, like point B in the figure. At these two points, the box's kinetic energy is equivalent, even if the orientation of its velocity is different.

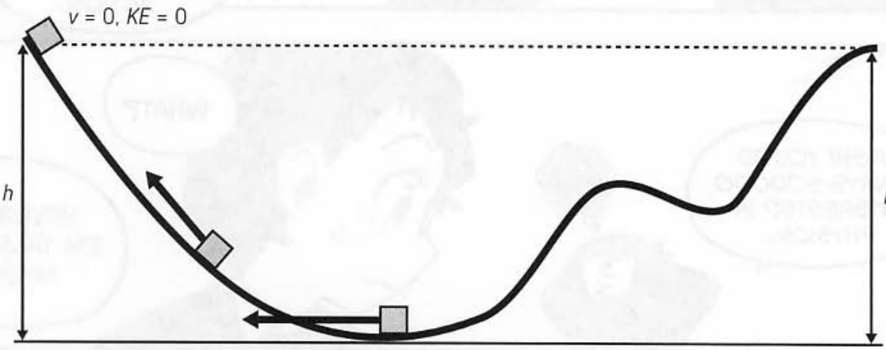


Kinetic energy is not associated with the orientation of velocity!

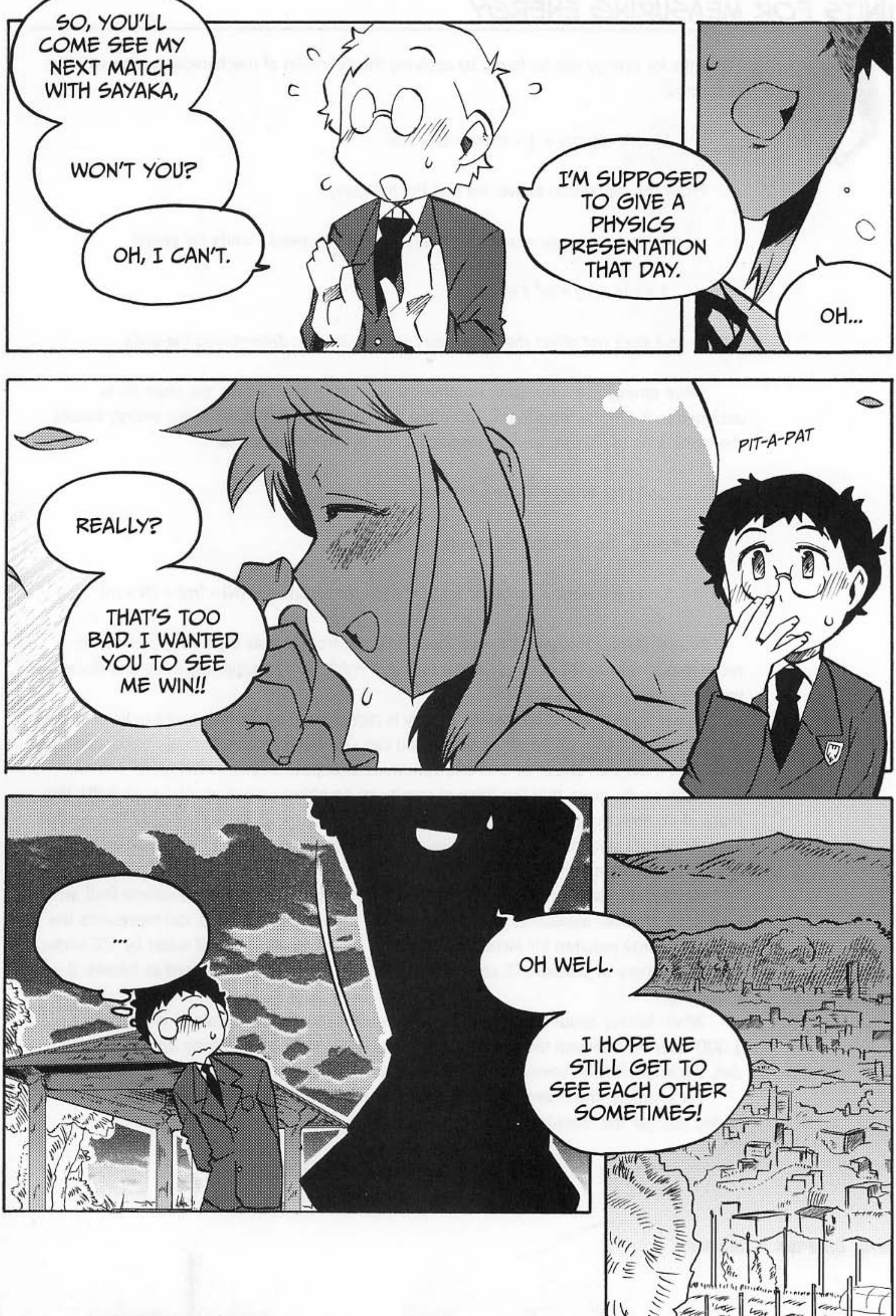
Yes, sir! Er, ma'am. Kinetic energy only has a magnitude. Similarly, potential energy only depends on height.

If we extended this slope, would it be possible for the box to go back up to its original height again?

Yes, it would be possible, provided that friction and air resistance are negligible. Of course, it'd be impossible to go beyond that original height of h.




YOUR LAST DIAGRAM MAKES ME FEEL LIKE RIDING A ROLLER COASTER.
OH YES, AN AMUSEMENT PARK
IS A TREASURE TROVE OF PHYSICS IN ACTION.
WHAT?
ARGH! YOU'RE ALWAYS SOOOOO INTERESTED IN PHYSICS...
NEVERMIND. I'M TALKING TO MYSELF.
NOW...
I THINK WE'VE COVERED THE BASICS OF THE LAWS OF MECHANICS.
I'M GLAD YOU STUCK WITH IT.
THANK YOU.
WITH YOUR HELP, I'VE STARTED TO UNDERSTAND WHAT PHYSICS IS ALL ABOUT.
GREAT! AND IF YOU'VE COME TO BE FOND OF PHYSICS, EVEN IF JUST A LITTLE...
I'M GLAD TOO.


SO, YOU'LL COME SEE MY NEXT MATCH WITH SAYAKA,
WON'T YOU?
OH, I CAN'T.
I'M SUPPOSED TO GIVE A PHYSICS PRESENTATION THAT DAY.
OH...
REALLY?
THAT'S TOO BAD. I WANTED YOU TO SEE ME WIN!!
PIT-A-PAT
...
OH WELL.
I HOPE WE STILL GET TO SEE EACH OTHER SOMETIMES!

BRING IT!
NO MATTER HOW POWERFUL HER SMASH IS...
VELOCITY AFTER STRIKING
FORCE
VELOCITY BEFORE STRIKING
IMPULSE GIVEN BY THE RACKET
MOMENTUM AFTER STRIKING
MOMENTUM AFTER STRIKING
THE RELATIONSHIP BETWEEN MOMENTUM AND FORCE...
DETERMINES THE VELOCITY OF MY RETURN.
SO...
SQUEAK


THE VELOCITY ALSO DETERMINES ITS SUBSEQUENT MOTION!
YOINKS!
OH, THAT'S VERY OBSERVANT OF YOU.
RYOTA TAUGHT YOU WELL!
BUT RYOTA...
HE DIDN'T MAKE IT TODAY, DID HE?
YOU KNOW EVERYTHING YOU NEED TO KNOW. ALL YOU HAVE TO DO TO WIN IS CONCENTRATE!

OOPS!
ビクッ
TUM
PANT PANT...
タッ タッ
ADVANTAGE, SAYAKA.
タッ タッ
NOT YET, YOU DON'T.
NINOMI...
ME—MEGU!!
PANT PANT...
HEY!
NOW,
I'VE GOT YOU, MEGUMI.

I MADE IT...
RYOTA!?!?!
YOU'RE HERE!!
...
I ASKED THE ORGANIZERS TO DELAY MY PRESENTATION.
TEE-HEE! THAT'S GREAT.
MEGU, REMEMBER, JUST CONCENTRATE ON THE GAME.
ALL RIGHT!
I CAN WIN THIS THING. IT'S MY SERVE.


HEY, YOU FINALLY CALLED ME MEGU!
WHAT'S GOING ON BETWEEN THESE TWO WEIRDOS?
IMPULSE
MOMENTUM
SMASH THE BALL!
DESTROY IT, CRUSH IT!


ACE!
THAT WAS SO FAST.
SHE'S SO MUCH BETTER!
I WONDER WHO HER NEW TENNIS INSTRUCTOR IS!!
AGAIN!!


ACE! ADVANTAGE TO MEGUMI!
WAY TO GO.
LET'S DO THIS THING. JUST ONE MORE SERVE.
ONE MORE TIME.
I CAN'T GO DOWN LIKE THIS.
I'M NUMBER ONE.
I CAN DO THIS!!
ゴクッ
CONCENTRATE, SAYAKA!!


I REMEMBER YOUR LESSONS PERFECTLY, RYOTA.
MAKE MY BODY FLEXIBLE.
MAXIMIZE FORCE WHEN THE RACKET STRIKES THE BALL!


ACE!
GAME, SET, MATCH!
WON BY MEGUMI!
I...
I DID IT! I WON, RYOTA!
HUH?!


HEY, YOU!
YOU CAN'T GET AWAY FROM ME, YOU KNOW.
UM...
AHEM. WILL YOU BE MY PARTNER FOR THE NEXT DOUBLES MATCH?
...
SURE, IT'S A DONE DEAL.
あははは
YOU KNOW RYOTA,
THIS FEELS LIKE A FORCE OF ATTRACTION.
PERHAPS IT IS...
WHAT ARE YOU TWO TALKING ABOUT?!